Unit 1 : Statics

* Resultant of two forces meeting at apoint.

let F_1, F_2 and R are the magnitudes of $\vec{F_1}$, $\vec{F_2}$ and $\vec{R}$

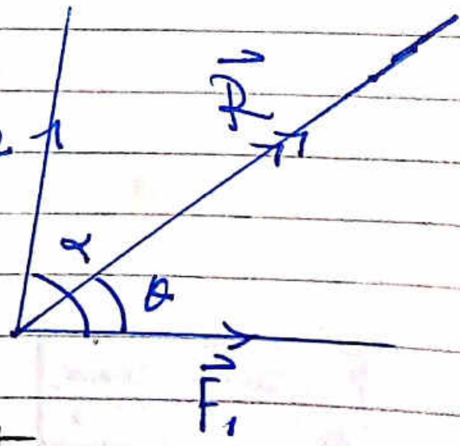
then:

$$\boxed{R = \sqrt{(F_1)^2 + (F_2)^2 + 2 F_1 F_2 \cos \alpha}}$$

$$\boxed{\tan \theta = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}}$$

where:

α is the measure of angle between $\vec{F_1}$, $\vec{F_2}$



and:

θ is the angle between the Resultant $\vec{R}$ and the force $\vec{F_1}$

* Special Cases:

1 If the two forces are perpendicular:

i.e $\alpha = 90^\circ$

$$\boxed{R = \sqrt{F_1^2 + F_2^2}}$$

and

$$\boxed{\tan\theta = \frac{F_2}{F_1}}$$

2 If the two forces are equal in magnitude.

i.e. $F_1 = F_2$

$$\boxed{R = 2F\cos\frac{\alpha}{2}}$$

$$\boxed{\theta = \frac{\alpha}{2}}$$

we note that:

$\vec{R}$ bisects the angle between the two forces

3 If the resultant is perpendicular to the first force

i.e $\theta = 90^\circ$

$$\boxed{R = \sqrt{F_2^2 - F_1^2}}$$

$$\boxed{F_1 + F_2 \cos \alpha = 0}$$

we note that

$F_1 < F_2$ and α is an obtuse angle

i.e R is perpendicular to the smallest force.

4 If the two forces have the same line of action and the same direction

i.e. $\alpha = 0^\circ$

$$\boxed{R = F_1 + F_2}$$

* R has the same direction of F_1, F_2

* R is called the greatest or the maximum value of the resultant

5) If the two forces have the same line of action but in opposite directions.

i.e. $\alpha = 180^\circ$

$$\boxed{R = |F_1 - F_2|}$$

* R has the direction of the greatest force in magnitude.

* R is called the smallest or the minimum value of the resultant.

6) If the two forces are equal in magnitude and have the same line of action but in opposite direction

$$\boxed{R = \text{zero}}$$

Example 1:

Two forces are of magnitudes 3 and 4 newton, Find:

(1) The maximum value of their resultant
(2) The minimum value of their resultant
(3) The magnitude and the direction of the resultant if the measure of the included angle between their direction $= 120^\circ$

Solution

(1) $R = F_1 + F_2 = 3 + 4 = 7$ newton

(2) $R = |F_1 - F_2| = |3 - 4| = 1$ newton

(3) $F_1 = 3$ newton , $F_2 = 4$ newton , $\alpha = 120^\circ$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$= \sqrt{9 + 16 + 2 \times 3 \times 4 \times \cos 120} = \sqrt{13} \text{ newton}$$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2\cos\alpha} = \frac{4\sin 120}{3 + 4\cos 120} = 2\sqrt{3}$$

$$\Rightarrow \theta = 73^\circ\, 53'\, 52''$$

Example 2:

Two forces of magnitudes $8\sqrt{3}$, 8 newton act at at a particle and enclose between them an angle of measure 150°. Find the magnitude of their resultant and the measure of the angle which it makes with the first force.

Solution:

$F_1 = 8\sqrt{3}$ newton , $F_2 = 8$ newton

, $\alpha = 150$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$R = \sqrt{(8\sqrt{3})^2 + (8)^2 + 2\times(8\sqrt{3})\times 8\times\cos 150}$$

$= 8$ newton.

$$\tan\theta = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta} = \frac{8\sin 150}{8\sqrt{3} + 8\cos 150} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \theta = 30°$

Example 3:

Two forces are of magnitude 8 and 16 gm.wt. acting at a particle. Find the measure of the angle included between the two directions of the two forces if the resultant is ~~perpendi~~ perpendicular to the first force.

Solution:

$F_1 = 8 \quad , F_2 = 16$ gm. wt.

$\because$ R is perpendicular to F_1

$\therefore F_1 + F_2 \cos\alpha = 0 \Rightarrow \cos\alpha = \frac{-F_1}{F_2}$

$\Rightarrow \cos\alpha = -\frac{8}{16} = -\frac{1}{2} \Rightarrow \alpha = 120^\circ$

Example 4: Two forces are of magnitude 30 and 16 newton act at a particle, if magnitude of their result is 26 newton. find the measure of angle between these two forces.

Solution: $R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\alpha$

$\Rightarrow (26)^2 = (30)^2 + (16)^2 + 2 \times 30 \times 16 \times \cos\alpha$

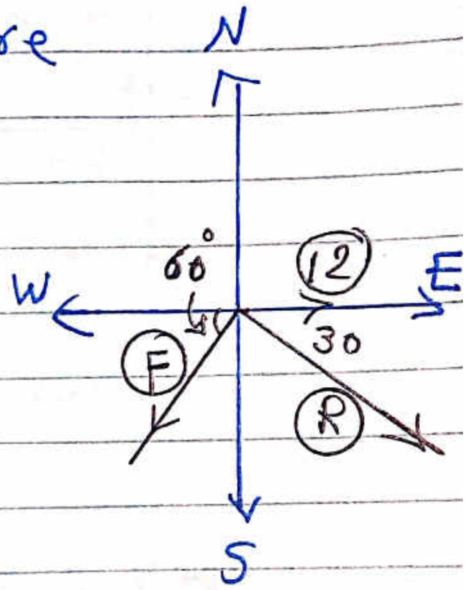
$\Rightarrow \cos\alpha = \frac{(26)^2 - (30)^2 - (16)^2}{2 \times 30 \times 16} = -\frac{1}{2}$

$\Rightarrow \alpha = 120^\circ$

Example 5: Two forces of magnitudes 12, F kg.wt. act on a point. The first force act in direction of east and the second force acts in direction 60° south of the west. Find the magnitude of F and the magnitude of the resultant if it is known that the line of action of the resultant acts in the direction 30° south of the east.

Solution

from the opposite figure



$F_1 = 12$, $F_2 = F$

$\alpha = 120°$, $\theta = 30°$

$$\Rightarrow \tan 30 = \frac{F \sin 120}{12 + F \cos 120}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\frac{\sqrt{3}}{2} F}{12 + (-\frac{1}{2})F} \Rightarrow \frac{3}{2} F = 12 - \frac{1}{2} F$$

$$\Rightarrow 2F = 12$$

$$\Rightarrow F = 6 \text{ kg.wt.}$$

$$R = \sqrt{(12)^2 + (6)^2 + 2 \times 12 \times 6 \times \cos 120}$$

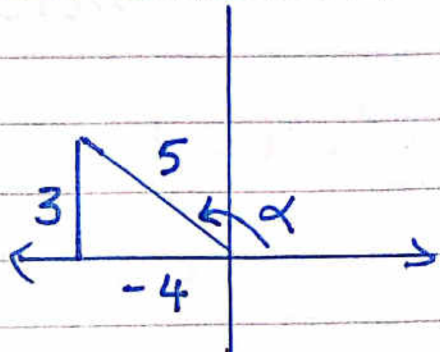
$$= 6\sqrt{3} \text{ kg.wt.}$$

Example 6:

Two forces of magnitudes 12 and 15 newton act on a particle and the cosine of the angle between them equals $-\frac{4}{5}$ find the magnitude of their resultant and the measure of the angle included between the resultant and the first force.

Solution:

$F_1 = 12$, $F_2 = 15$ newton



$\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{-4}{5}$

note that α is an obtuse angle

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

$$= \sqrt{(12)^2 + (15)^2 + 2 \times 12 \times 15 \times \frac{-4}{5}}$$

$$= 9 \text{ newton.}$$

$$, \tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha} = \frac{15 \times \frac{3}{5}}{12 + 15 \times -\frac{4}{5}} = \frac{9}{0}$$

$$\Rightarrow \cot\theta = 0 \quad \Rightarrow \theta = 90^\circ$$

Example 7

two forces of magnitudes F, 4 newton act on a particle and the measure of the angle between their directions is 120°, the magnitude of their resultant equals $4\sqrt{3}$ newton, Find the magnitude of $\vec{F}$ and the measure of the angle that $\vec{R}$ from with $\vec{F}$

Solution: $R = 4\sqrt{3}$ newton

$F_1 = F$, $F_2 = 4$ newton , $\alpha = 120^\circ$

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha$$

$$(4\sqrt{3})^2 = F^2 + (4)^2 + 2 \times F \times 4 \times \cos 120$$

$$48 = F^2 + 16 - 4F$$

$$\Rightarrow F^2 - 4F - 32 = 0$$

$$(F - 8)(F + 4) = 0$$

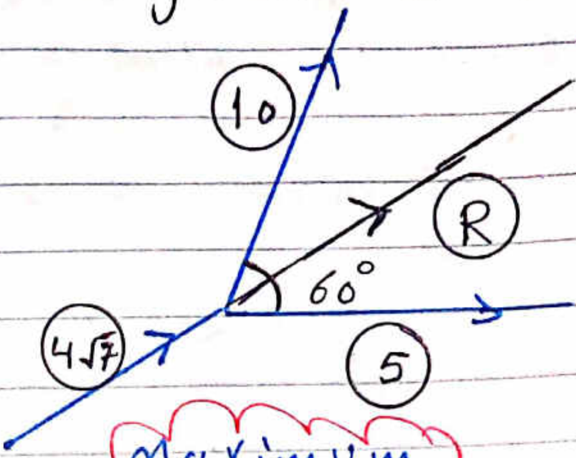
$$\Rightarrow F = 8 \text{ newton}$$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2\cos\alpha} = \frac{4\sin 120}{8 + 4\cos 120} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^\circ$$

Example 8:

Three forces of magnitude 5, 10, $4\sqrt{7}$ N act on a particle, if the measure of the angle between the first and the second forces equals 60°, find the magnitude of the maximum and the minimum resultant for the three forces.



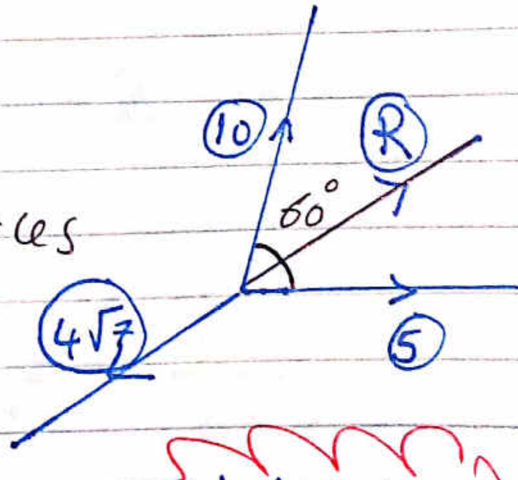
Solution

First we will find the resultant of the first and the second forces

Maximum

$$R = \sqrt{(5)^2 + (10)^2 + 2 \times 5 \times 10 \cos 60}$$

$$= 5\sqrt{7} \text{ N}$$



maximum result of 3 forces

$$= 5\sqrt{7} + 4\sqrt{7}$$

$$= 9\sqrt{7} \text{ N}$$

minimum

minimum resultant of the three forces

$$= |5\sqrt{7} - 4\sqrt{7}| = \sqrt{7} \text{ N}$$

Example 9: two forces of magnitudes 2, F newton, the angle between them is of measure 120° Find F in each of the following cases:

(1) magnitude of the resultant equals F
(2) The direction of the resultant is perpendicular to the second force
(3) the Resultant bisects the angle between the two forces.

Solution: $F_1 = 2$, $F_2 = F$, $\alpha = 120°$

(1) $R = F$

$$\therefore R^2 = 2^2 + F^2 + 2 \times F \times 2 \cos 120°$$

$$\Rightarrow \cancel{R^2} = 4 + \cancel{F^2} - 2F$$

$$2F = 4 \Rightarrow F = 2 \text{ newton}$$

(2) $\theta = 30°$

$$\tan\theta = \frac{F_2 \sin\alpha}{F_1 + F_2\cos\alpha} \Rightarrow \frac{1}{\sqrt{3}} = \frac{F \sin 120}{2 + F\cos 120}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\frac{\sqrt{3}}{2}F}{2 - \frac{1}{2}F} \Rightarrow F = 1 \text{ newton}$$

(3) $F_1 = F_2 = 2$

because R bisects the angle between the two forces

Example 10:

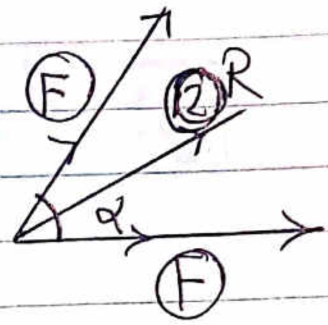
magnitude

two forces of equal magnitude, magnitude of their resultant equals 12 kg.wt. if the direction of one of them is reversed then the magnitude of the resultant becomes 6 kg.wt. Find the magnitude of each force.

Solution:

$F_1 = F_2 = F \quad , \quad R = 12$ kg.wt.

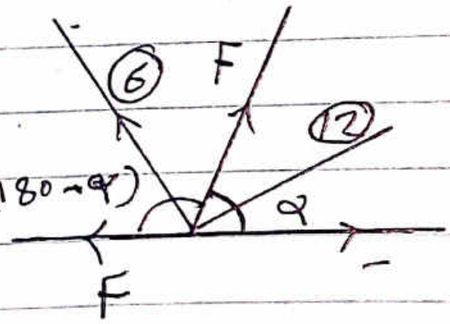
First Case:



$$R = 2F\cos\frac{\alpha}{2}$$

$$12 = 2F\cos\frac{\alpha}{2} \quad \cdots (1)$$

Second Case:

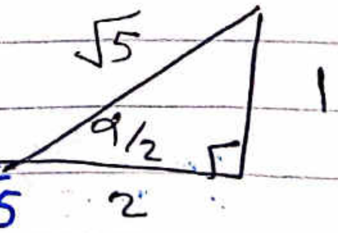


$$6 = 2F\cos\left(\frac{180-\alpha}{2}\right)$$

$$6 = 2F\cos\left(90 - \frac{\alpha}{2}\right)$$

$$\Rightarrow 6 = 2F\sin\frac{\alpha}{2} \quad \cdots (2)$$

from (2) ÷ (1)



$$\frac{1}{2} = \tan\frac{\alpha}{2} \quad , \quad \cos\frac{\alpha}{2} = \frac{2}{\sqrt{5}}$$

substitute in (1)

$$F = 3\sqrt{5} \text{ kg.wt.}$$

Example 11: two forces of same magnitude F kg.wt. enclose between them an angle of measure $120°$ If the two forces are doubled and the measure of the angle between them became $60°$, then the magnitude of their resultant increase by 11 kg.wt. than the first case. Find the magnitude of F

Solution

First case:

$$F_1 = F_2 = F$$

$$\Rightarrow R = 2F \cos \frac{\alpha}{2}$$

$$R = 2F \cos 60 \Rightarrow R = F$$

Second case:

$$F_1 = F_2 = 2F \quad , R = F + 11$$

$$\Rightarrow F + 11 = 2 \times 2F \cos 30$$

$$F + 11 = 2\sqrt{3}\, F$$

$$\Rightarrow (2\sqrt{3} - 1) F = 11$$

$$F = \frac{11}{2\sqrt{3} - 1} = 1 + 2\sqrt{3}$$

Example 12: $F, 2F$ are two forces act on a point and enclose between them an angle of measure α the magnitude of their resultant equals $\sqrt{5}\,F(m+1)$ and if the measure of the angle between them becomes $(90^\circ-\alpha)$ then the magnitude of the resultant will be $\sqrt{5}\,F(m-1)$ prove that:

$$\tan\alpha = \frac{m-2}{m+2}$$

Solution:

In the first case

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\alpha ,$$

$$\Rightarrow \left[\sqrt{5}\,F(m+1)\right]^2 = F^2 + 4F^2 + 4F^2\cos\alpha$$

$$\Rightarrow 5F^2(m+1)^2 = 5F^2 + 4F^2\cos\alpha$$

$$5F^2(m^2+2m+1) = 5F^2 + 4F^2\cos\alpha$$

$$5F^2m^2 + 10F^2m + \cancel{5F^2} = \cancel{5F^2} + 4F^2\cos\alpha$$

$$\cancel{F^2}(5m^2+10m) = 4\cancel{F^2}\cos\alpha$$

$$\Rightarrow 5m^2 + 10m = 4\cos\alpha \quad \dots (1)$$

from the second case:

$$\left[\sqrt{5}\,F(m-1)\right]^2 = F^2 + 4F^2 + 4F^2\cos(90-\alpha)$$

$$5F^2(m^2-2m+1)=5F^2+4F^2\sin\alpha$$

$$5F^2m^2-10F^2m+\cancel{5F^2}=\cancel{5F^2}+4F^2\sin\alpha$$

$$\cancel{F^2}(5m^2-10m)=4\cancel{F^2}\sin\alpha$$

$$4\sin\alpha=5m^2-10m \quad \text{---} \ (2)$$

from (2) ÷ (1)

$$\frac{\cancel{4}\sin\alpha}{\cancel{4}\cos\alpha}=\frac{5m^2-10m}{5m^2+10m}$$

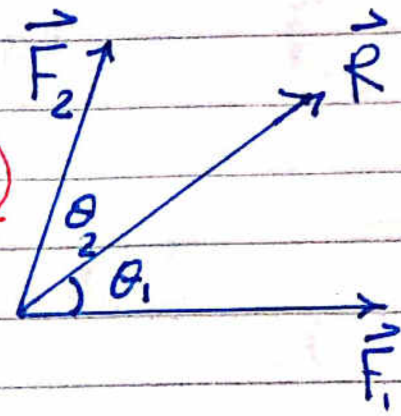
$$\Rightarrow \tan\alpha=\frac{5m(m-2)}{5m(m+2)}$$

$$\Rightarrow \tan\alpha=\frac{m-2}{m+2}$$

Lesson .2 :

Forces resolution into two Components.

$$\frac{F_1}{\sin\theta_2} = \frac{F_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1+\theta_2)}$$



where :

* F_1 the magnitude of the Component of $\vec{R}$ which inclines by θ_1 on $\vec{R}$
* F_2 the magnitude of the Component of $\vec{R}$ which inclines by θ_2 on $\vec{R}$

Example: A force of magnitude 600 kg.wt. acts on a particle. Find its two components on two directions making with the force two angles of measure 30° and 45°

Solution

$$\frac{F_1}{\sin\theta_2} = \frac{F_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1+\theta_2)}$$

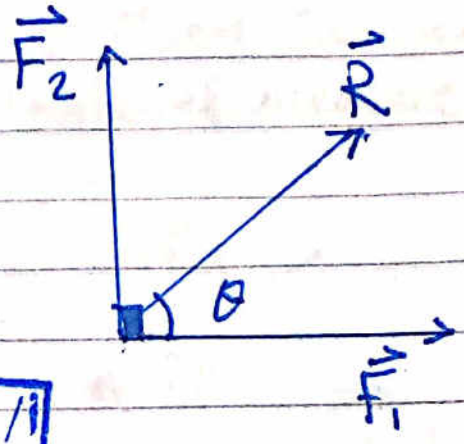
$$\Rightarrow \frac{F_1}{\sin 45} = \frac{F_2}{\sin 30} = \frac{600}{\sin 75}$$

$$\Rightarrow F_1 = 439.23 \text{ kg.wt.} , F_2 = 310.58 \text{ kg.wt}$$

Resolution of the force into two perpendicular directions:

$$F_1 = R \cos\theta$$

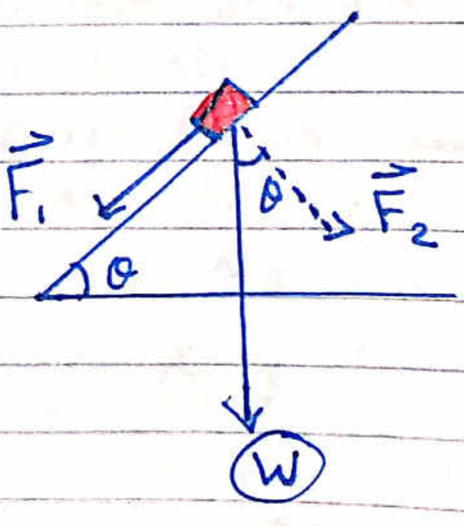
$$F_2 = R \sin\theta$$



If a body of weight (w) is placed on a smooth inclined plane with the horizontal by an angle θ then:

$$F_1 = w \sin\theta$$

$$F_2 = w \cos\theta$$



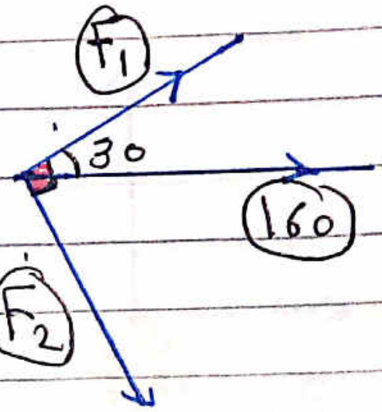
where:

* F_1 is the magnitude of the component in the direction of the line of the greatest slope

* F_2 is the mgnitude of the component in the perpendicular direction on the plane

Example 2: Resolve a horizontal force of magnitude 160 gm.wt. in two perpendicular directions one of them inclined to the horizontal with an angle of measure 30° upwards.

Solution

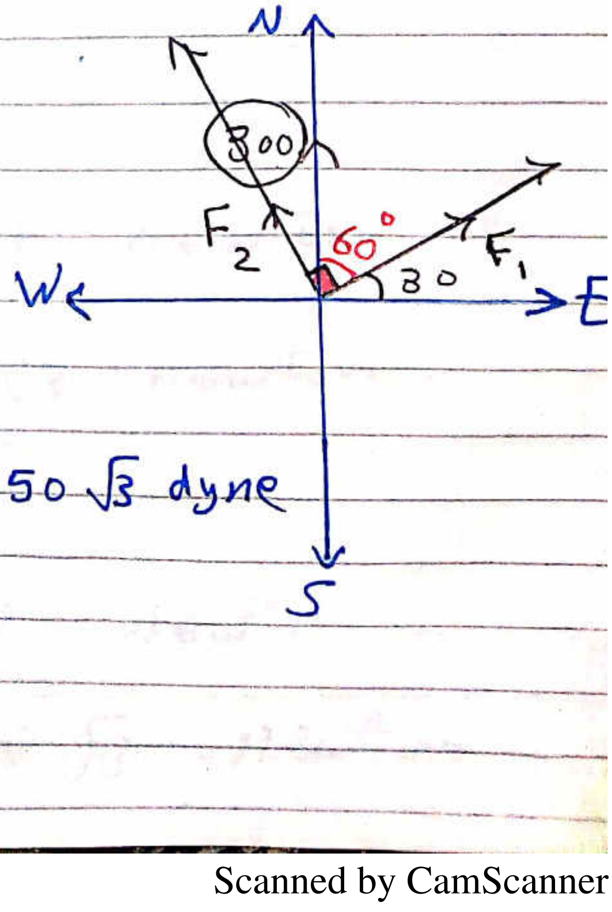


$F_1 = 160 \cos 30 = 80\sqrt{3}$ gm.wt.

$F_2 = 160 \sin 30 = 80$ gm.wt.

Example 3: A force of magnitude 300 dyne acts in the north direction. Find the magnitude of the two perpendicular component if one of them acts in the direction 30° North of East.

Solution:

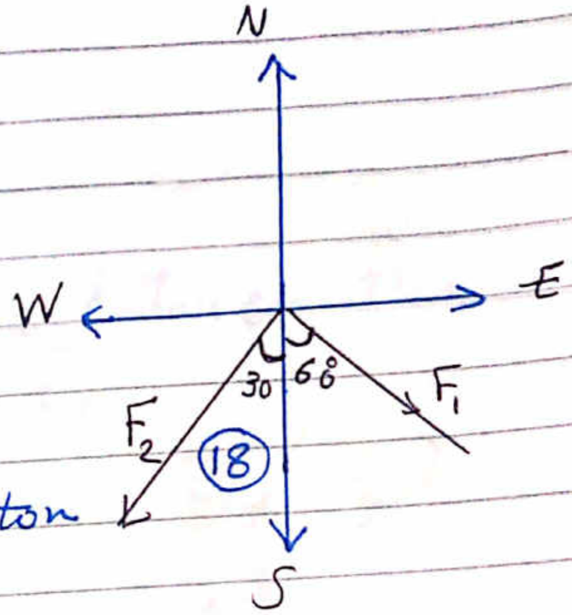


$F_1 = 300 \cos 60$
$= 150$ dyne

$F_2 = 300 \sin 60 = 150\sqrt{3}$ dyne

Example 4: A Force of magnitude 18 newton acts in the direction of South. Find its two Components in the two direction 60° East of the South and the other direction towards 30° West of the South.

Solution:



$$\frac{F_1}{\sin 30} = \frac{F_2}{\sin 60} = \frac{18}{\sin 90}$$

$$\Rightarrow F_1 = \frac{18 \sin 30}{\sin 90} = 9 \text{ newton}$$

$$\& F_2 = \frac{18 \sin 60}{\sin 90} = 9\sqrt{3} \text{ newton}$$

Another Solution

$F_1 = 18 \cos 60 = 9$ newton

$F_2 = 18 \sin 60 = 9\sqrt{3}$ newton

third Solution:

$F_1 = 18 \sin 30 = 9$ newton

$F_2 = 18 \cos 30 = 9\sqrt{3}$ newton

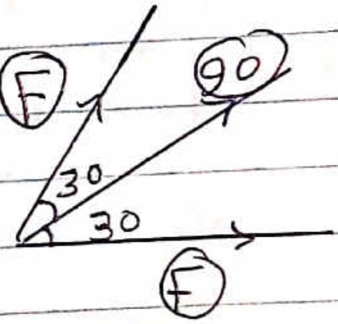
Example 5:

Resolve of force of magnitude 90 newton into two equal forces in magnitude and the measure of the angle between their lines of action is $60°$.

Solution

$\because F_1 = F_2$

$\therefore \vec{R}$ bisects the angle between the lines action of $\vec{F_1}$, $\vec{F_2}$



$$\therefore \frac{F}{\sin 30} = \frac{F}{\sin 30} = \frac{90}{\sin 60}$$

$$\therefore F = \frac{90 \sin 30}{\sin 60} = 30\sqrt{3} \text{ newton}$$

Another Solution

$$\because R = 2F\cos\frac{\alpha}{2}$$

$$\therefore 90 = 2F\cos 30$$

$$F = \frac{90}{2\cos 30} = 30\sqrt{3} \text{ newton}$$

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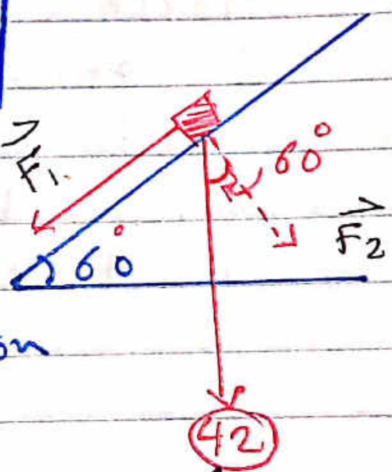
Example 6:

A rigid body of weight 42 newton is placed on a plane inclined to the horizotal with an angle of measure 60°. Find the two components of weight of the body in the direction of the line of the greatest slope and the direction normal to it.

Solution

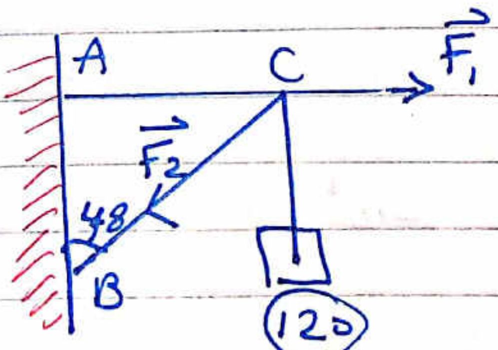


$F_1 = 42 \sin 60$
$= 21\sqrt{3}$ newton

$F_2 = 42 \cos 60 = 21$ newton

Example 7

Find F_1, F_2



Solution:

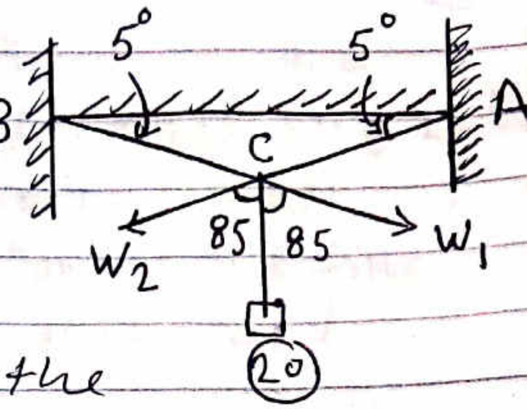
$$\frac{F}{\sin 48} = \frac{F_2}{\sin 90} = \frac{120}{\sin(90+48)}$$

$$F_1 = \frac{120 \sin 48}{\sin 138} = 133.27 \text{ gm.wt.}$$

$$F_2 = \frac{120 \sin 90}{\sin 138} = 179.34 \text{ gm.wt.}$$

Example 8

In the opposite figure:



① Resolve the weight 20 N into two component in the directions $\overrightarrow{AC}$, $\overrightarrow{BC}$

Approximating the result to the nearest newton

② what happens to the magnitude of the Components of the weight in the directions of the metal rods if the measure of the inclimation angle to the horizontal decreased to be smaller than 5°? And what do you expect to the components when the rods become horizontal?

Solution

$$\frac{w_1}{\sin 85} = \frac{w_2}{\sin 85} = \frac{20}{\sin(85+85)}$$

$$\Rightarrow w_1 = w_2 = \frac{20 \sin 85}{\sin 170} = 115 \text{ newton}$$

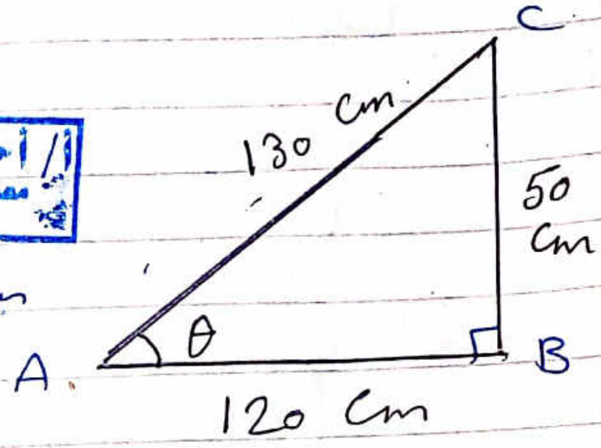
(2) If the measure of the angle decreased with horizontal less than 5°, then the magnitude of the Component will increase

(3) when the rods became horizontal the Components will become unlimited becose $\sin 180 = 0$

$$w_1 = w_2 = \frac{20 \sin 90}{\sin 180} = \infty$$

Example 9

An inclined plane of length 130 Cm. and height 50 Cm. a rigid body of weight 390 gm.wt is placed on it. Find the two components of the weight in the direction of the line of greatest slope of the plane and perpendicular to it.



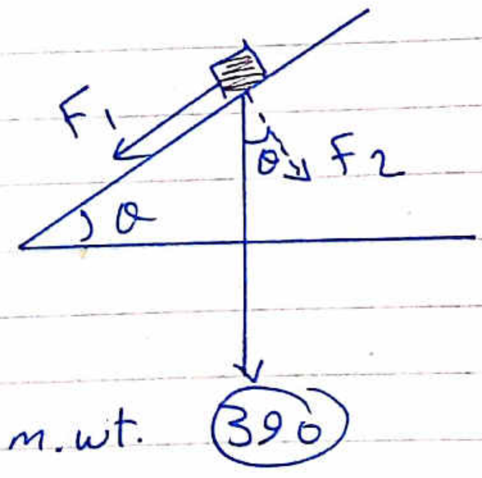
Solution

from pythagora's theorem ~~Since~~ :

$AB = 120$ Cm

$\Rightarrow \sin\theta = \frac{50}{130} = \frac{5}{13}$

$\cos\theta = \frac{120}{130} = \frac{12}{13}$



$F_1 = 390 \sin\theta$

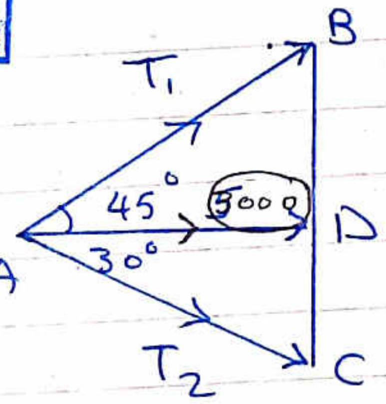
$= 390 \times \frac{5}{13} = 150$ gm.wt.

$F_2 = 390 \cos\theta = 390 \times \frac{12}{13} = 360$ gm.wt

Example 10

A cruiser is pulled by two ships B and C. using two stands hanged to apoint A on the cruiser, the measure of the angle' between the two stands equals $75°$, if the angle between one of the stands and the $\overrightarrow{AD}$ equals $45°$ and the resultant of the forces used to pull the cruiser equals 5000 newton and acts on $\overrightarrow{AD}$.
Find the tension in the two stands.

Solution:



$$\frac{T_1}{\sin\theta_2} = \frac{T_2}{\sin\theta_1} = \frac{R}{\sin(\theta_1+\theta_2)}$$

$$\frac{T_1}{\sin 30} = \frac{T_2}{\sin 45} = \frac{5000}{\sin 75}$$

$$T_1 = \frac{5000\sin 30}{\sin 75} \simeq 2588.2 \text{ newton}$$

$$T_2 = \frac{5000\sin 45}{\sin 75} \simeq 3660.3 \text{ newton}$$

Lesson 3:

The resultant of Coplanar forces meeting at apoint.

Suppose that the system of Coplanar forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_N$ meet at the point O and the point O is the origin point of a Coplanar Cartisian axis.

and $\theta_1, \theta_2, \theta_3, \dots, \theta_n$ are the polar angles of the forces respectively

then

$$\vec{R} = X\,\vec{i} + Y\,\vec{j}$$

where:

$$X = \sum_{r=1}^{n} F_r \cos \theta_r$$

$$Y = \sum_{r=1}^{n} F_r \sin \theta_r$$

and

$$R = \sqrt{X^2 + Y^2}$$

$$\tan \alpha = \frac{Y}{X}$$

To determine the direction of the resultant :

X	Y	quad.	θ
+	+	1^{st}	measure of the acute angle
−	+	2^{nd}	180 − measure of the acute angle
−	−	3^{rd}	180 + measure of the acute
+	−	4^{th}	360° − measure of the acute angle

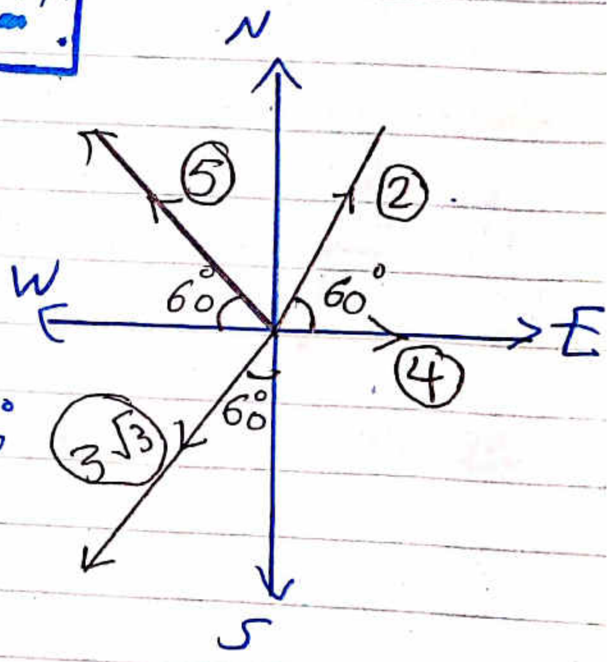
Example 1:

Four Coplanar forces act on a particle the first of magnitude 4 newton acts in the Eastern direction, the second of magnitude 2 newton, acts in direction 60° North of the East, the third of magnitude 5 newton acts in direction 60° north of the west and the fourth of magnitude $3\sqrt{3}$ newton acts in direction 60° West of the south.

Find the magnitude and direction of their resultant

Solution:

F	4	2	5	$3\sqrt{3}$
θ	0°	60°	120°	210°



$$X = 4\cos 0 + 2\cos 60^\circ + 5\cos 120^\circ + 3\sqrt{3}\cos 210^\circ = -2$$

$$Y = 4\sin 0^\circ + 2\sin 60^\circ + 5\sin 120^\circ + 3\sqrt{3}\sin 210 = 2\sqrt{3}$$

$$\vec{R} = -2\,\vec{i} + 2\sqrt{3}\,\vec{j}$$

$$R = \sqrt{(-2)^2 + (2\sqrt{3})^2} = 4 \text{ newton}$$

$$\tan\alpha = \frac{Y}{X} = \frac{2\sqrt{3}}{-2} = -\sqrt{3} \Rightarrow \theta = 180 - 60 = 120$$

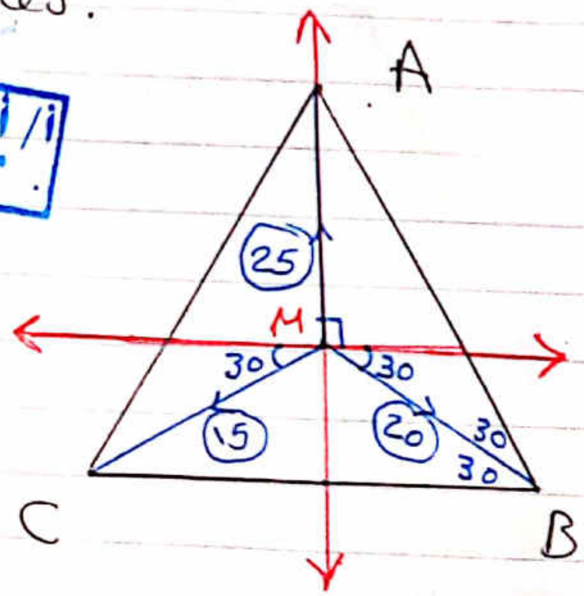
$\because X < 0,\ Y > 0 \quad \therefore \theta$ lies in 2^{nd} quad

Example 2:

ABC is an equilateral triangle. M is the point of intersection of its medians. the forces of magnitude 15, 20 and 25 newton act on a particle at the point M in the direction of $\overrightarrow{MC}$, $\overrightarrow{MB}$, $\overrightarrow{MA}$

Find the magnitude and the direction of the resultant of these forces.

Solution



F	25	15	20
θ	90°	210°	330°

$$X = 25\cos 90^\circ + 15\cos 210^\circ + 20\cos 330^\circ$$

$$= \frac{5}{2}\sqrt{3} \text{ newton}$$

$$Y = 25\sin 90^\circ + 15\sin 210^\circ + 20\sin 330^\circ$$

$$= \frac{15}{2}$$

$$\vec{R} = \frac{5}{2}\sqrt{3}\,\vec{i} + \frac{15}{2}\,\vec{j}$$

$$R = \sqrt{\left(\frac{5}{2}\sqrt{3}\right)^2 + \left(\frac{15}{2}\right)^2} = 5\sqrt{3} \text{ newton}$$

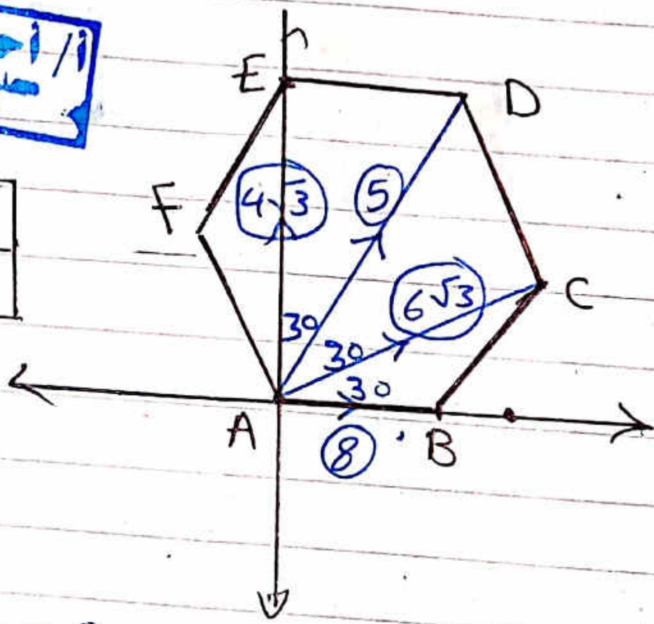
$$\tan\alpha = \frac{Y}{X} = \sqrt{3}, \quad X>0, \ Y>0 \ \therefore \theta = 60^\circ$$

Example 3:

ABCDEF is a regular hexagon, the forces of magnitudes $8, 6\sqrt{3}, 5, 4\sqrt{3}$ newton act on $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant.

Solution

F	8	$6\sqrt{3}$	5	$4\sqrt{3}$
θ	$0°$	$30°$	$60°$	$90°$



$$X = 8\cos 0° + 6\sqrt{3}\cos 30° + 5\cos 60 + 4\sqrt{3}\cos 90$$
$$= 19.5 \text{ newton}$$

$$Y = 8\sin 0° + 6\sqrt{3}\sin 30° + 5\sin 60° + 4\sqrt{3}\sin 90°$$
$$= \frac{19}{2}\sqrt{3}$$

$$\vec{R} = \frac{39}{2}\vec{i} + \frac{19}{2}\sqrt{3}\,\vec{j}$$

$$R = \sqrt{\left(\frac{39}{2}\right)^2 + \left(\frac{19}{2}\sqrt{3}\right)^2} = \sqrt{651} \text{ newton}$$

$\because X > 0, Y > 0 \quad \therefore \theta$ lies in 1st quad.

$$\tan\theta = \frac{Y}{X} = \frac{19\sqrt{3}}{39} \Rightarrow \theta = 40° 9' 30''$$

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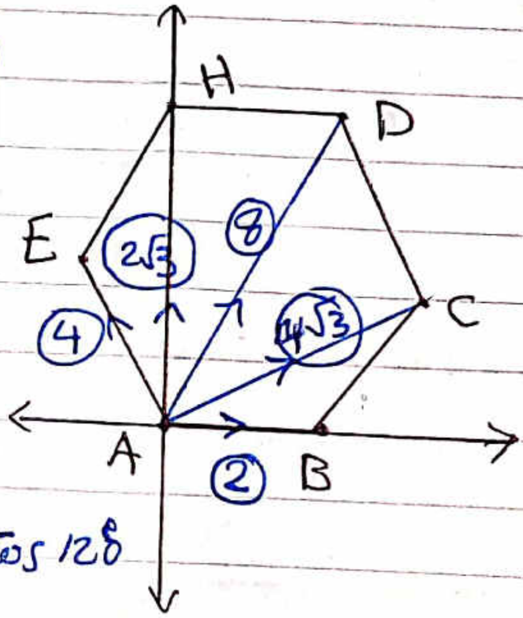
Example 4

ABCDHE is a regular hexagon, forces of magnitudes $2, 4\sqrt{3}, 8, 2\sqrt{3}$ and 4 kg.wt. act at point A in directions $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AH}$, $\overrightarrow{AE}$ respectively. Find the magnitude and the direction of their resultant.

Solution



F	2	$4\sqrt{3}$	8	$2\sqrt{3}$	4
θ	0°	30°	60°	90°	120°

$$X = 2\cos 0^\circ + 4\sqrt{3}\cos 30^\circ + 8\cos 60^\circ + 2\sqrt{3}\cos 90^\circ + 4\cos 120 = 10 \text{ kg.wt.}$$

$$Y = 2\sin 0^\circ + 4\sqrt{3}\sin 30^\circ + 8\sin 60^\circ + 2\sqrt{3}\sin 90^\circ + 4\sin 120 = 10\sqrt{3} \text{ kg.wt.}$$

$$\vec{R} = 10\,\vec{i} + 10\sqrt{3}\,\vec{j}$$

$$R = \sqrt{(10)^2 + (10\sqrt{3})^2} = 20 \text{ kg.wt}$$

$\because X > 0,\ Y > 0 \quad \therefore \theta$ lies in 1^{st} quad.

$$\tan\theta = \frac{Y}{X} = \sqrt{3}$$

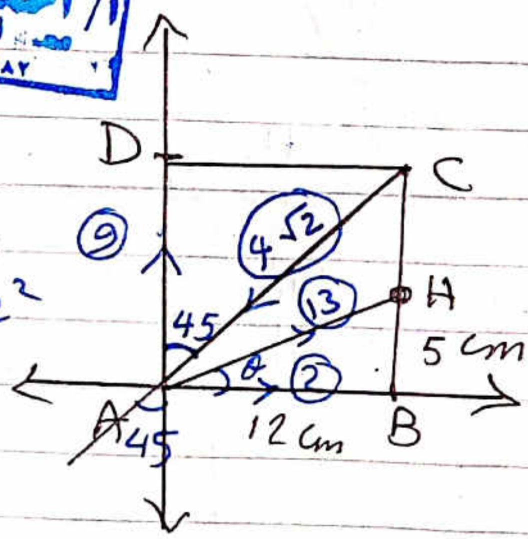
$$\therefore \theta = 60^\circ$$

i.e R in the direction of $\overrightarrow{AD}$

Example 5

ABCD is a square of side length is 12 cm $H \in \overline{BC}$ where BH = 5 cm. forces of magnitudes 2, 13, $4\sqrt{2}$, 9 gm.wt. act in directions of $\overrightarrow{AB}$, $\overrightarrow{AH}$, $\overrightarrow{CA}$ and $\overrightarrow{AD}$ respectively. Find the magnitude of the resultant of these forces.

Solution:



F	2	13	$4\sqrt{2}$	9
θ	0°	θ	225°	90°

from $\triangle ABH \Rightarrow (AH)^2 = 5^2 + 12^2$

$\Rightarrow AH = 13$ cm

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$X = 2\cos 0^\circ + 13\cos\theta + 4\sqrt{2}\cos 225 + 9\cos 90^\circ$

$= 2 + 13 \times \frac{12}{13} + (-4) + 0 = 10$ gm.wt.

$Y = 2\sin 0^\circ + 13\sin\theta + 4\sqrt{2}\sin 225 + 9\sin 90^\circ$

$= 0 + 13 \times \frac{5}{13} + (-4) + 9 = 10$ gm.wt.

$R = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ gm.wt.

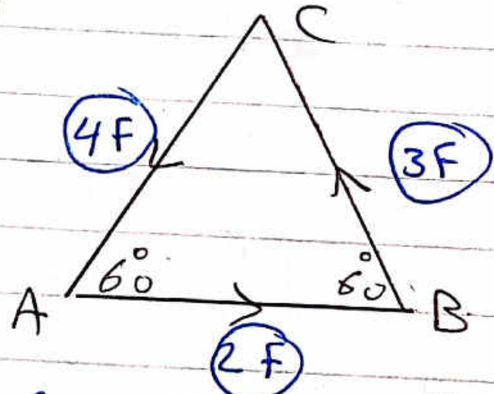
$\tan\alpha = \frac{Y}{X} = \frac{10}{10} = 1 \Rightarrow \alpha = 45^\circ$

$\therefore \vec{R}$ in direction of $\overrightarrow{AC}$

Example 6:

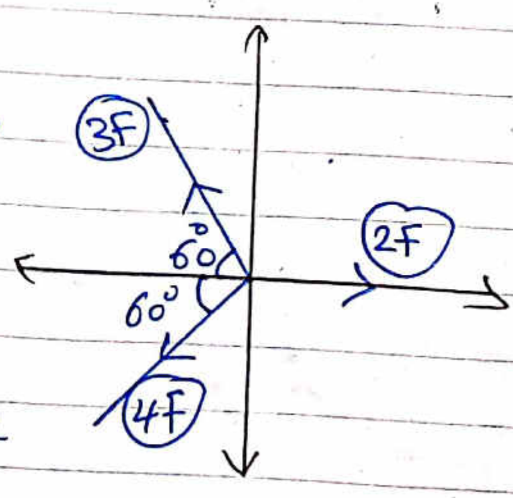
The forces of magnitudes 2F, 3F and 4F newton act on a particle in the directions parallel to the sides of an equilateral triangle in the same cyclic order. Find the magnitude and the direction of the resultant of these forces.

Solution:



F	2F	3F	4F
θ	0°	120°	240°

$$X = 2F\cos 0° + 3F\cos 120° + 4F\cos 240°$$
$$= -\frac{3}{2}F \text{ newton}$$



$$Y = 2F\sin 0 + 3F\cos 120 + 4F\sin 240°$$
$$= -\frac{\sqrt{3}}{2}F \text{ newton}$$

$$R = \sqrt{\left(-\frac{3}{2}F\right)^2 + \left(-\frac{\sqrt{3}}{2}F\right)^2}$$
$$= \sqrt{3}F \text{ newton}$$

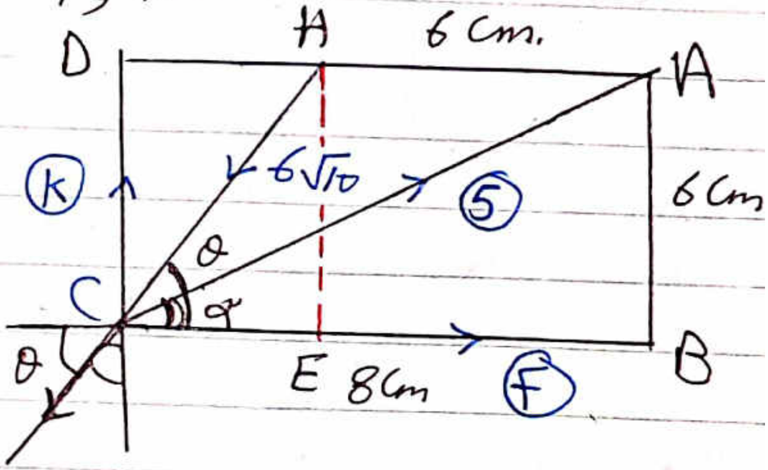
$\because X < 0,\ Y < 0 \quad \therefore \theta$ lies in 3^{rd} quad.

$$\therefore \tan\alpha = \frac{Y}{X} = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 180 + 30 = 210°$$

Example 7: In the opposite figure:
the forces of magnitudes F, 5, k and $6\sqrt{10}$ N are in equilibrium and they act in the rectangle ABCD in the directions $\overrightarrow{CB}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{HC}$. Such that:
AB = 6 cm, BC = 8 cm, AH = 6 cm.
Find the value of F, K

Solution



In Δ ABC:

$(AC)^2 = 6^2 + 8^2 = 100$

$\Rightarrow AC = 10$ cm

$\Rightarrow \sin\alpha = \frac{6}{10} = \frac{3}{5}$, $\cos\alpha = \frac{8}{10} = \frac{4}{5}$

In Δ CEH:

$\because$ EH = 6 cm, CE = 2 cm

$\therefore (CH)^2 = 6^2 + 2^2 = 40 \Rightarrow CH = 2\sqrt{10}$ cm

$\cos\theta = \frac{\cancel{2}}{\cancel{2}\sqrt{10}} = \frac{1}{\sqrt{10}}$, $\sin\theta = \frac{\cancel{6}}{2\sqrt{10}} = \frac{3}{\sqrt{10}}$

$\Rightarrow$

Force	F	5	$6\sqrt{10}$	K
Polar angle	0°	α	$180+\theta$	90°

$$X = F\cos 0^\circ + 5\cos\alpha + 6\sqrt{10}\cos(180+\theta) + K\cos 90^\circ$$

$$= F + 5\times\frac{4}{5} + 6\sqrt{10}\left(-\frac{1}{\sqrt{10}}\right) + 0 \;\cancel{=18-18}$$

$$= F + 4 - 6 = F - 2$$

$$\because X = 0 \Rightarrow F - 2 = 0 \Rightarrow \boxed{F = 2}$$

$$Y = F\sin 0^\circ + 5\sin\alpha + 6\sqrt{10}\sin(180+\theta) + K\sin 90^\circ$$

$$= 0 + 5\times\frac{3}{5} + 6\sqrt{10}\times\left(-\frac{3}{\sqrt{10}}\right) + K$$

$$= 3 - 18 + K = -15 + K$$

$$\because Y = 0 \Rightarrow -15 + K = 0$$

$$\Rightarrow \boxed{K = 15}$$

__note:__

If the forces in equilibrium

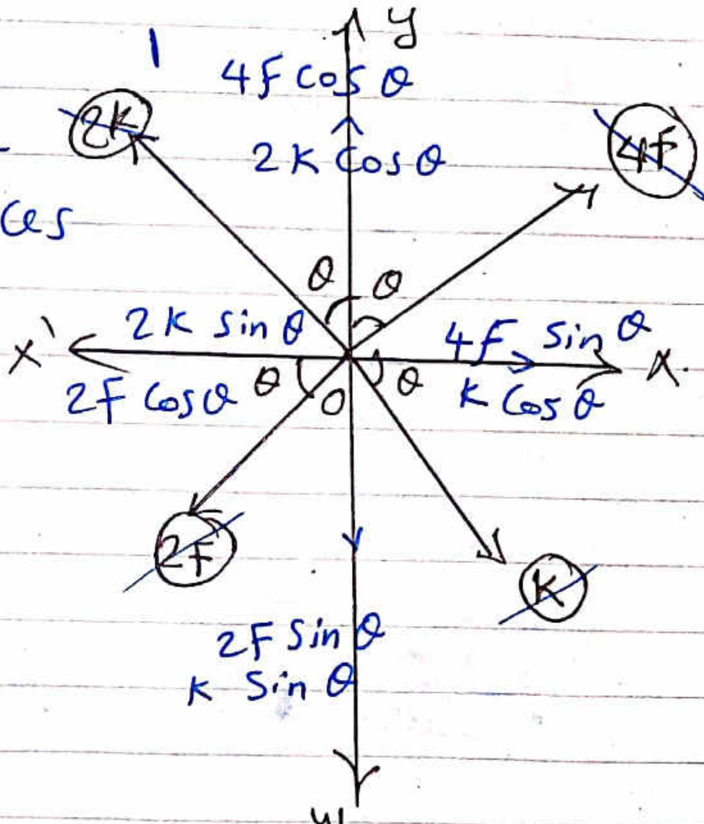
$$\therefore R = 0 \Rightarrow X = 0,\; Y = 0$$

Example 8: In the opposite figure:

Four coplanar forces act at the point (O) in the directions shown in the figure where $\sin\theta = \frac{4}{5}$ and the resultant of these forces is $8\sqrt{2}$ N and makes an angle of measure 135° with $\overrightarrow{OX}$, then find the values of F and K

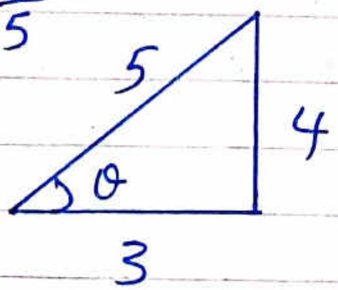
Solution

We will solve by using the analyzing of the forces into two perpendicular directions.



$\because \sin\theta = \frac{4}{5}$

$\therefore \cos\theta = \frac{3}{5}$



$X = 4F\sin\theta + K\cos\theta - 2K\sin\theta - 2F\cos\theta$

$= 4F \times \frac{4}{5} + K \times \frac{3}{5} - 2K \times \frac{4}{5} - 2F \times \frac{3}{5}$

$= 2F - K \quad \ldots \; (1)$

$Y = 4F\cos\theta + 2K\cos\theta - 2F\sin\theta + K\sin\theta$

$= 4F \times \frac{3}{5} + 2K \times \frac{3}{5} - 2F \times \frac{4}{5} + K \times \frac{4}{5}$

$\Rightarrow$

$$Y = \frac{4}{5} F + \frac{2}{5} K \quad \ldots (2)$$

$\because \vec{R} = (8\sqrt{2}, 135^\circ)$

$\therefore \vec{R} = 8\sqrt{2} \cos 135 \, \vec{i} + 8\sqrt{2} \sin 135 \, \vec{j}$

$= -8 \, \vec{i} + 8 \, \vec{j} \quad \ldots (3)$

$\therefore 2F - K = -8 \quad \ldots (A)$

$, \ \frac{4}{5} F + \frac{2}{5} K = 8 \quad \ldots (B)$

by solving A, B

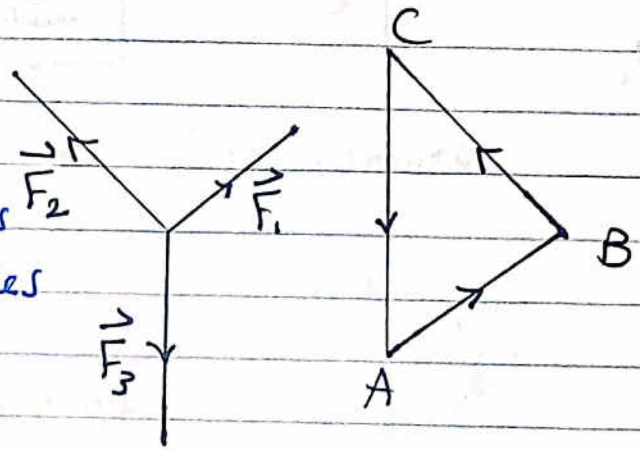
$\therefore F = 3$ Newton

, $K = 14$ newton

Lesson 4: Equilibrium of a rigid body under the effect of two forces / Three forces meeting at a point

The triangle of forces rule:

If ΔABC is drawn whose sides are parallel to the lines of action of the forces and taken in the same cyclic order then



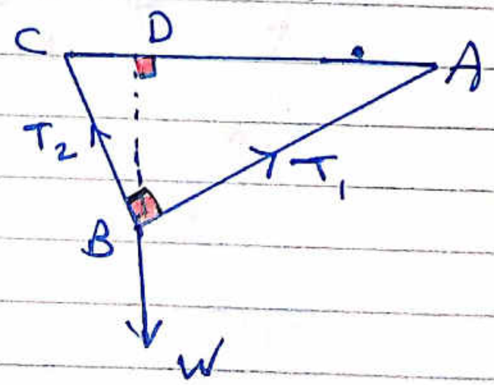
$$\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{F_3}{AC}$$

ΔABC is called " the triangle of forces"

* The rule of perpendicular forces triangle

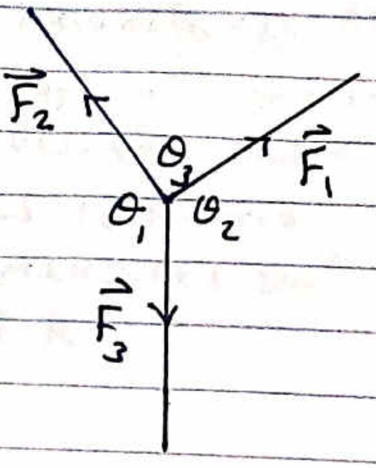
If $\vec{W} \perp \overline{AC}$, $\vec{T_1} \perp \overline{BC}$, $\vec{T_2} \perp \overline{AB}$

$$\therefore \frac{W}{AC} = \frac{T_1}{BC} = \frac{T_2}{AB}$$



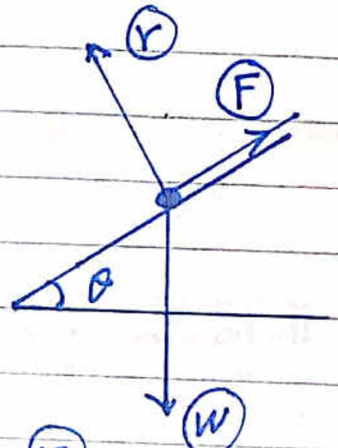
Lami's rule:

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

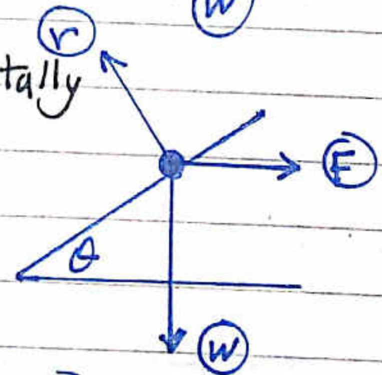


Equilibrium of abody placed on asmoth inclined plane:

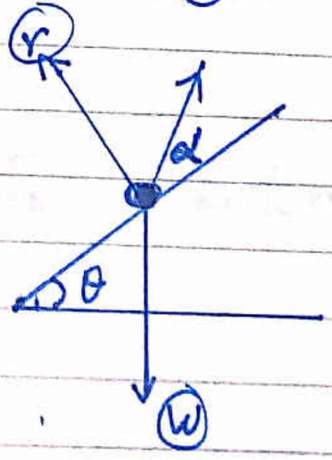
1. The force in the direction of the line of the greatest slope.



2. The force acts horizontally



3. The force inclines by α ~~witha~~ with the plane upwards.



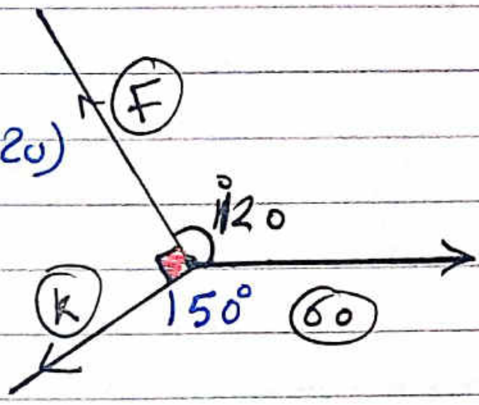
Exampl ①

Three coplanar forces of magnitudes 60, F, K newton meeting at apoint and in equilibrium. If the angle between the 1st and the 2nd force measures 120° and between the 2nd and 3rd measures 90° Find the value of F and k.

Solution

the measure of the third angle = 360 − (90 + 120)

= 150°



from lami's rule:

$$\frac{F}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

$$\frac{60}{\sin 90} = \frac{F}{\sin 150} = \frac{k}{\sin 120}$$

$$F = \frac{60 \sin 150}{\sin 90} = 30 \text{ newton}$$

$$k = \frac{60 \sin 120}{\sin 90} = 30\sqrt{3} \text{ newton}$$

Example 2

A particle is in equilibrium under effect of three forces of magnitudes F_1, F_2 and 75 newton they are represented by the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ of ΔABC respectively where : $AB = 3$ cm., $BC = 4$ cm. and $CA = 5$ cm. Find the value of F_1 and F_2

Solution

by The triangle of forces rule:

$$\frac{F_1}{AB} = \frac{F_2}{BC} = \frac{75}{AC} \Rightarrow \frac{F_1}{3} = \frac{F_2}{4} = \frac{75}{5}$$

$$\Rightarrow F_1 = \frac{3 \times 75}{5} = 45 \text{ newton}$$

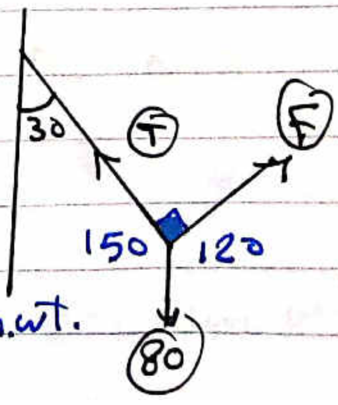
$$, F_2 = \frac{4 \times 75}{5} = 60 \text{ newton.}$$

Example ③

A lamp of weight 80 gm.wt. is attached to the end of a string. It is in equilibrium under the effect of a force perpendicular to the string when it is inclined to the vertically by an angle of measure 60°. Find F, T.

Solution

from lami's rule:



$$\frac{T}{\sin 120} = \frac{F}{\sin 150} = \frac{80}{\sin 90}$$

$$\Rightarrow F = 40 \text{ gm.wt}, T = 40\sqrt{3} \text{ gm.wt.}$$

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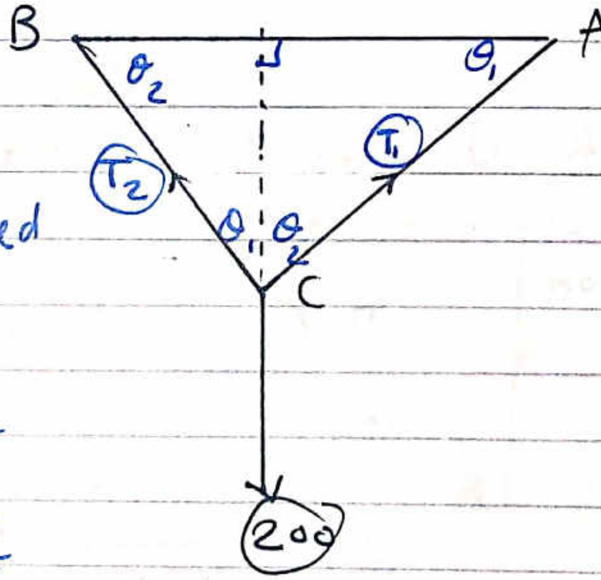
Example 4: A weight of magnitude 200 gm.wt. is suspended by two strings of length 60 cm and 80 cm., from two points on one horizontal line where the distance between them is 100 cm. Find the magnitude of tension in each string.

Solution:



$\because (60)^2 + (80)^2 = (100)^2$

$\therefore \Delta ABC$ is right angled at C

$$\sin\theta_1 = \frac{BC}{AB} = \frac{60}{100} = \frac{3}{5}$$

$$\sin\theta_2 = \frac{AC}{AB} = \frac{80}{100} = \frac{4}{5}$$

$$\therefore \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2} = \frac{200}{\sin(\theta_1+\theta_2)}$$

$$\Rightarrow \frac{T_1}{\frac{3}{5}} = \frac{T_2}{\frac{4}{5}} = \frac{200}{\sin 90} \Rightarrow \begin{cases} T_1 = 120 \text{ gm.wt} \\ T_2 = 160 \text{ gm.wt} \end{cases}$$

Another solution:

by perpendicular of forces triangle

$$\frac{T_1}{BC} = \frac{T_2}{AC} = \frac{200}{AB}$$

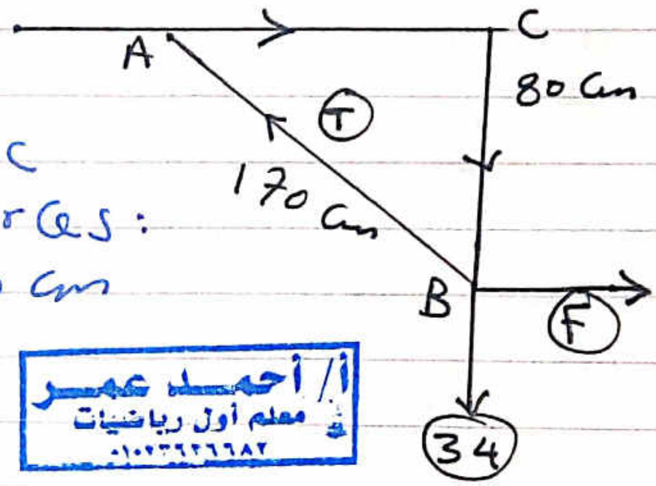
$$\Rightarrow \frac{T_1}{60} = \frac{T_2}{80} = \frac{200}{100} \Rightarrow \begin{cases} T_1 = 120 \text{ gm.wt} \\ T_2 = 160 \text{ gm wt} \end{cases}$$

5) A light string of length 170 cm. Its end A is fixed at a point of a ceilling of a room from the other end B there is a lamp of weight 34 gm.wt. Find the magnitude of each of the tension and the required force to make the lamp in equilibrium at a distance 80 cm. down the ceilling each of the following cases:

(1) If the force is horizotal.

(2) If the force is perpendicular to $\overline{AB}$

Solution:



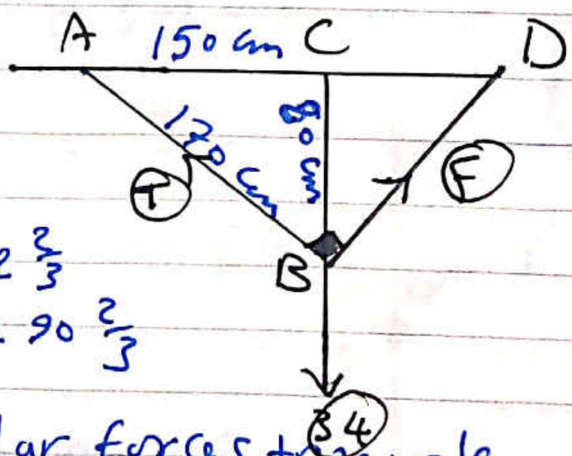
first case: take $\triangle ABC$ as the triangle of forces:

$AC = \sqrt{170^2 - 80^2} = 150$ cm

$\therefore \frac{T}{170} = \frac{F}{150} = \frac{34}{80}$

$\Rightarrow T = 72.25$ gm.wt , $F = 63.75$ gm.wt.

Second case:



$\triangle CDB \sim \triangle CBA$

$\Rightarrow \frac{CD}{CB} = \frac{DB}{BA} = \frac{BC}{AC}$

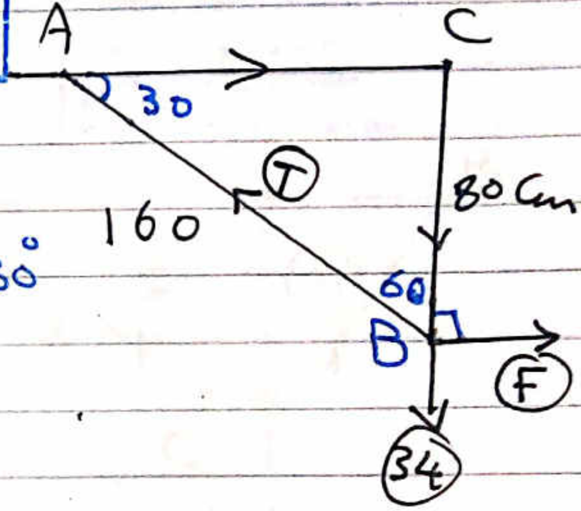
$\Rightarrow \frac{CD}{80} = \frac{DB}{170} = \frac{80}{150} \Rightarrow \begin{cases} CD = 42\frac{2}{3} \\ DB = 90\frac{2}{3} \end{cases}$

by using the perpendicular forces triangle.

$\frac{F}{AB} = \frac{T}{BD} = \frac{34}{AD} \Rightarrow \frac{F}{170} = \frac{T}{90\frac{2}{3}} = \frac{34}{192\frac{2}{3}} \Rightarrow \begin{cases} T = 16 \\ F = 30 \end{cases}$ gm.wt

6 In the previouse example if $AB = 160$ cm , $BC = 80$ cm
Find T, F

Solution



First Case:

$\because BC = \frac{1}{2} AB \quad \therefore m(\angle A) = 30^\circ$

$\therefore m(\angle ABC) = 60^\circ$

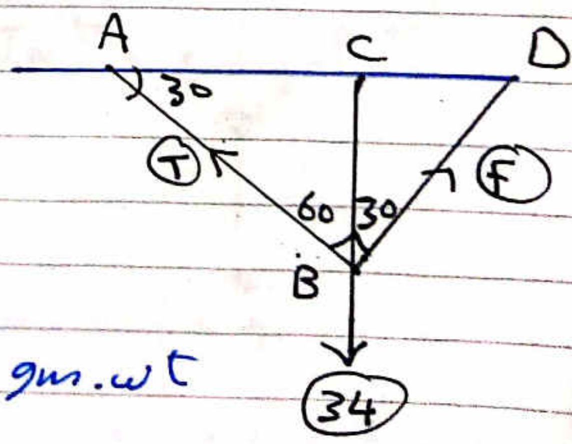
from lami's rule

$$\frac{F}{\sin 120} = \frac{T}{\sin 90} = \frac{34}{\sin 150}$$

$$\Rightarrow F = \frac{34 \sin 120}{\sin 150} = 34\sqrt{3} \text{ gm.wt}$$

$$T = \frac{34 \sin 90}{\sin 150} = 68 \text{ gm.wt.}$$

Second Case



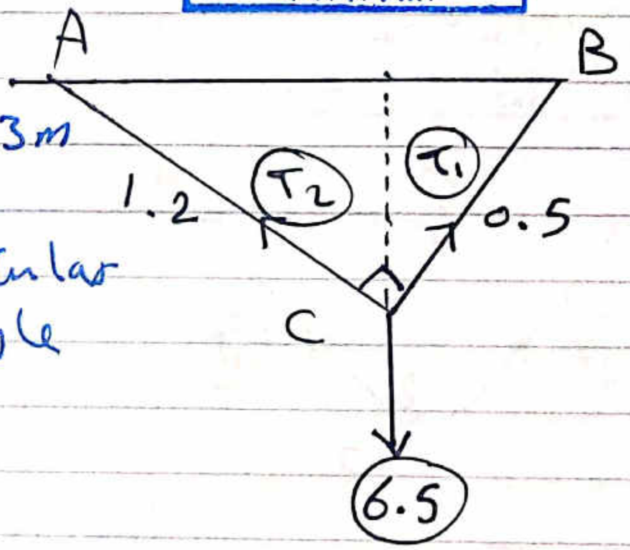
$$\frac{T}{\sin 30} = \frac{F}{\sin 60} = \frac{34}{\sin 90}$$

$$\Rightarrow T = \frac{34 \sin 30}{\sin 90} = 17 \text{ gm.wt}$$

$$, F = \frac{34 \sin 60}{\sin 90} = 17\sqrt{3} \text{ gm.wt.}$$

(7) A body of weight 6.5 newton is suspended by two strings of lengths 0.5 and 1.2 m the two other ends are fixed at two points on a horizontal line such that the strings are perpendicular to each other. Find the tension in each of the two strings.

Solution



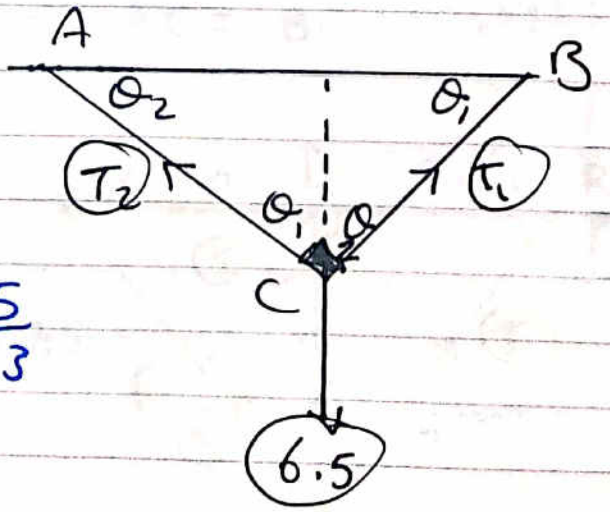
$AB = \sqrt{0.5^2 + 1.2^2} = 1.3\,m$

$\because \Delta ABC$ is perpendicular of forces triangle

$\therefore \frac{T_1}{1.2} = \frac{T_2}{0.5} = \frac{6.5}{1.3}$

$T_1 = 6$ newton , $T_2 = 2.5$ newton

Another solution :



$\sin\theta_1 = \frac{AC}{AB} = \frac{1.2}{1.3} = \frac{12}{13}$

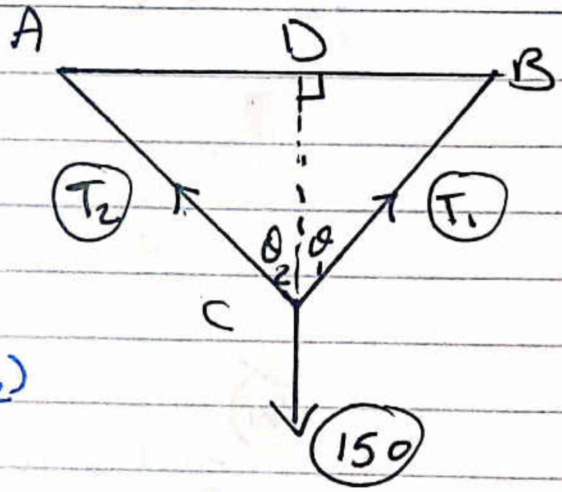
$\sin\theta_2 = \frac{BC}{AB} = \frac{0.5}{1.3} = \frac{5}{13}$

$\therefore \frac{T_1}{\sin\theta_1} = \frac{T_2}{\sin\theta_2} = \frac{6.5}{\sin 90}$

$\Leftrightarrow \frac{T_1}{\frac{12}{13}} = \frac{T_2}{\frac{5}{13}} = \frac{6.5}{1}$ $\Rightarrow$ $\left\{ \begin{array}{l} T_1 = 6.5 \times \frac{12}{13} = 6 \\ T_2 = 6.5 \times \frac{5}{13} = 2.5 \end{array} \right.$ newton

8 A smooth string of length 30 cm is attached by its end in the two points A, B such that $\overline{AB}$ is horizontally, AB = 18 cm. If a smooth ring of weight 150 gm.wt. slides on the strings prove that in the case of equilibrium the lengths of the two parts of the strings are equal, then find the tension in each part.

Solution



$\because$ The ring is smooth

$\therefore T_1 = T_2$

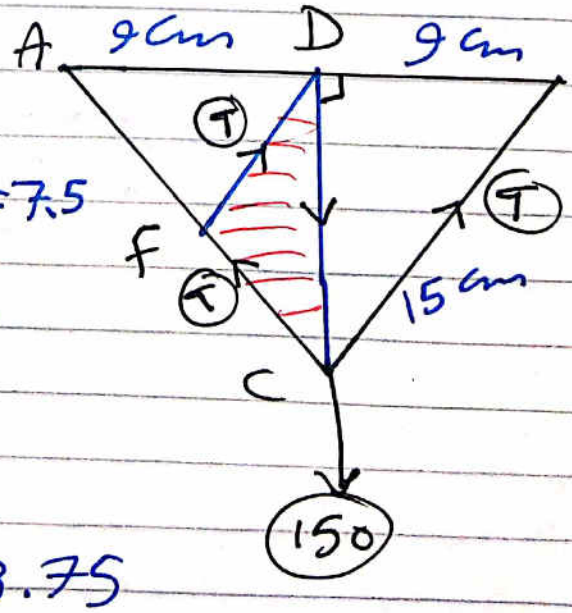
from lami's rule:

$$\frac{T}{\sin\theta_1} = \frac{T}{\sin\theta_2} = \frac{150}{\sin(\theta_1+\theta_2)}$$

$\therefore \sin\theta_1 = \sin\theta_2 \quad \therefore \theta_1 = \theta_2$

$\because \theta_1 = \theta_2,\ \overline{CD} \perp \overline{AB} \quad \therefore AC = BC$ (first)

from the figure



$CD = \sqrt{15^2 - 9^2} = 12$ cm

$FD = \frac{1}{2} BC = 7.5,\ FC = 7.5$

$\triangle FDC$ is the triangle of forces

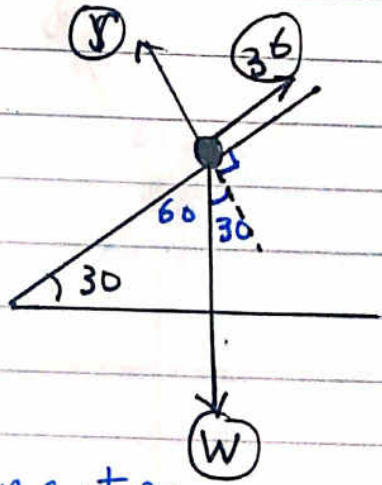
$$\frac{T}{7.5} = \frac{T}{7.5} = \frac{150}{12}$$

$$T = 7.5 \times \frac{150}{12} = 93.75$$

9

A body of weight (W) newton is placed on a smooth plane inclined with the horizotal at an angle of measure 30° and kept in equilibrium by the effect of force of magnitude 36 newton acts in the direction of the line of greatest slope of the plane upwards. Find the magnitude of the weight W and the magnitude of the reaction of the plane.

Solution:



from lami's rule

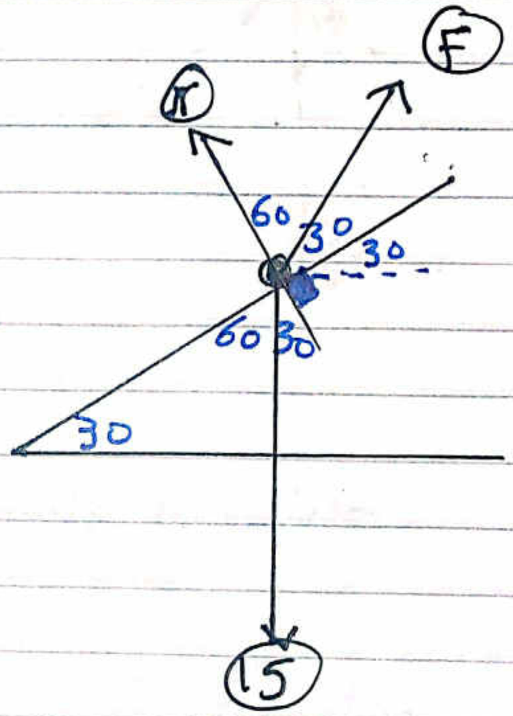
$$\frac{36}{\sin 150} = \frac{r}{\sin 120} = \frac{W}{\sin 90}$$

$$r = \frac{36 \sin 120}{\sin 150} = 36\sqrt{3} \text{ newton}$$

$$W = \frac{36 \sin 90}{\sin 150} = 72 \text{ newton}$$

10 A body of weight 15 kg.wt. is placed on a smooth plane inclines to the horizotal at an angle of measure $\sin^{-1}\frac{1}{2}$, a force inclined to the horizotal at an angle of measure 60 acted on the body to keep it in equilibrium. Find the magnitude of each of the force and the reaction of the plane.

Solution



from lami's rule:

$$\frac{r}{\sin 150} = \frac{F}{\sin 150} = \frac{15}{\sin 60}$$

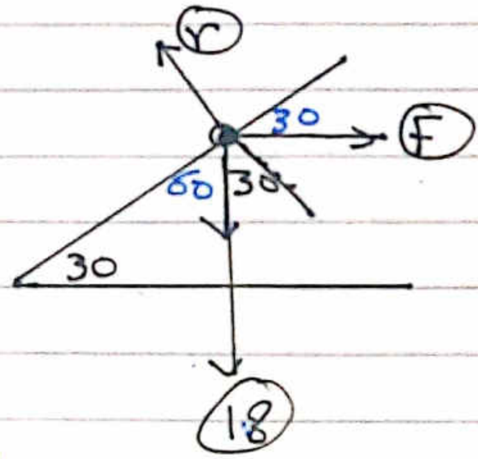
$$\Rightarrow r = F = \frac{15 \sin 150}{\sin 60}$$

$$\Rightarrow r = F = 5\sqrt{3} \text{ kg w.t}$$

note that

$$\theta = \sin^{-1}\frac{1}{2} = 30^\circ$$

11 A body of weight 18 newton is placed on a smooth plane inclined to the horizontal by an angle of measure 30°, It is kept in equilibrium by a horizontal Force F newton Find F, r



Solution

from lami's rule

$$\frac{r}{\sin 90} = \frac{F}{\sin 150} = \frac{18}{\sin 120}$$

$$r = \frac{18 \sin 90}{\sin 120} = 12\sqrt{3} \text{ newton}$$

$$F = \frac{18 \sin 150}{\sin 120} = 6\sqrt{3} \text{ newton}$$

Lesson 5: Follow: The equilibrium:

Rule 4

"If a rigid body is in equilibrium under the action of three coplanar non parallel forces, then the lines of action of these forces meet at a point"

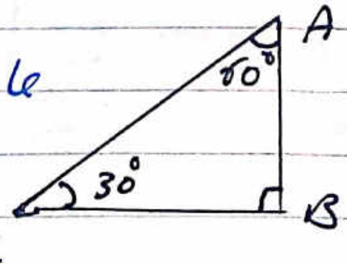
Remarks

1) The weight of the uniform sphere acts at its geometric centre

2) The weight of the rod (uniform rod) acts vertically downwards at its midpoint (centre of gravity)

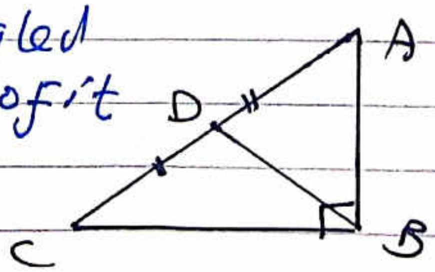
3) The reaction of the smooth vertically wall (R) is perpendicular to the wall

4) If ΔABC is $30^\circ - 60^\circ$ triangle



$\therefore AB = \frac{1}{2} AC$, $BC = \frac{\sqrt{3}}{2} AC$

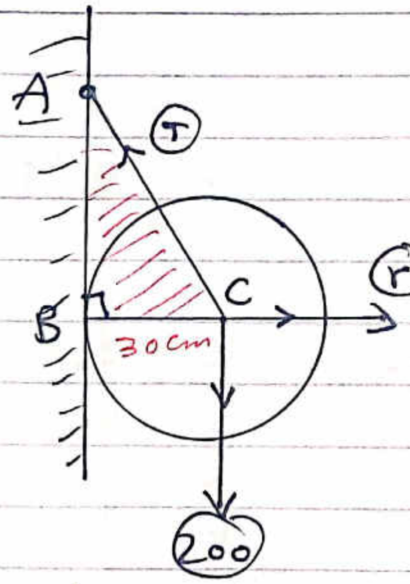
5) If ΔABC is a right-angled at B, $\overline{BD}$ is a median of it



then: $BD = \frac{1}{2} AC$

[1] A smooth sphere of radius length 30 cm. and of weight 200 gm.wt. rests on a vertical smooth wall. It is suspended by a string of length 20 cm., one of ends is attached to a point on the surface of the sphere and the other end is fixed at a point on the wall above the touch point of the sphere and the wall. Find the magnitude of the tension in the string and the reaction of the wall.

Solution



ΔABC is the triangle of forces:

$$\Rightarrow \frac{T}{AC} = \frac{r}{BC} = \frac{200}{AB}$$

$$\Rightarrow \frac{T}{50} = \frac{r}{30} = \frac{200}{40}$$

$$\Rightarrow T = \frac{200 \times 50}{40} = 250 \text{ gm.wt}$$

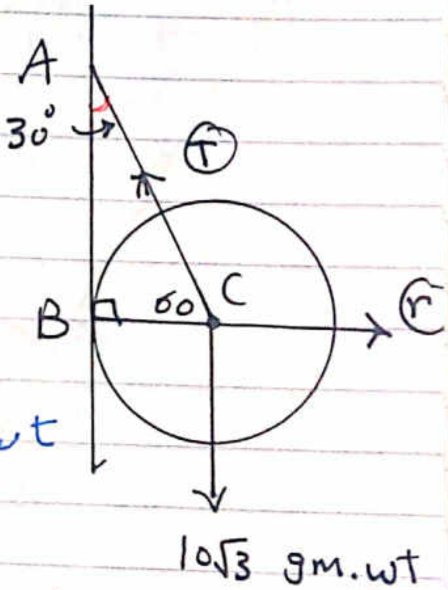
$$, r = \frac{30 \times 200}{40} = 150 \text{ gm.wt}$$

Note: that:

$$AB = \sqrt{(50)^2 - 30^2} = 40 \text{ cm}$$

2. A smooth sphere of weight $10\sqrt{3}$ gm.wt. rests against a smooth vertical wall. It is suspended at a point of its surface by means of a string and its other end is fixed to the wall at a point lies directly above the point of tangency of the sphere and the wall. If the string makes with the vertical an angle of measure $30°$. Find the tension in the string and the reaction of the wall

Solution



by using lami's rule:

$$\frac{T}{\sin 90} = \frac{r}{\sin 150} = \frac{10\sqrt{3}}{\sin 120}$$

$$\Rightarrow T = \frac{10\sqrt{3} \sin 90}{\sin 120} = 20 \text{ gm.wt}$$

$$, \ r = \frac{10\sqrt{3} \sin 150}{\sin 120} = 10 \text{ gm.wt}$$

Another solution: let $BC = \ell$

$\Rightarrow AC = 2\ell$, $AB = \sqrt{3}\ell$

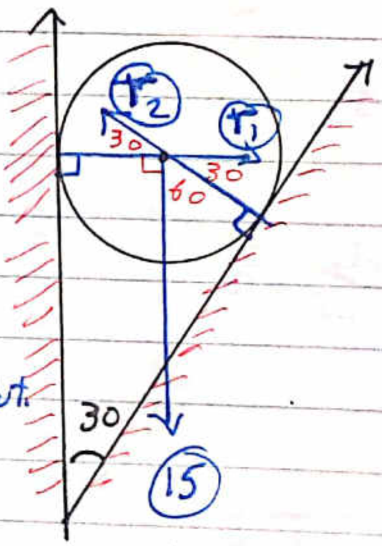
, ΔABC is the triangle of forces

$$\Rightarrow \frac{T}{2\ell} = \frac{r}{\ell} = \frac{10\sqrt{3}}{\sqrt{3}\ell} \Rightarrow \begin{cases} T = \dfrac{10\sqrt{3} \times 2\ell}{\sqrt{3}\ell} = 20 \text{ gm.wt.} \\ r = \dfrac{10\sqrt{3} \times \ell}{\sqrt{3}\ell} = 10 \text{ gm.wt} \end{cases}$$

3 A metalic sphere of weight 15 kg.wt is put such that it touches two smooth planes, one of them is vertical and the other inclines to the vertical by an angle of measure 30°. Find the ~~tension~~ reaction of each of the two planes.

Solution:

by using Lami's rule:



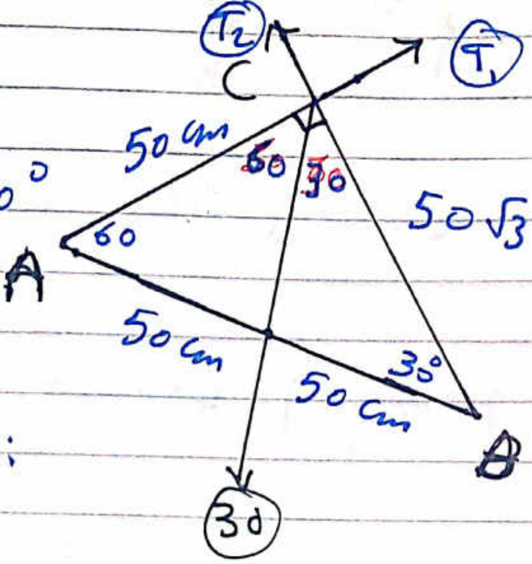
$$\frac{r_1}{\sin 120} = \frac{r_2}{\sin 90} = \frac{15}{\sin 150}$$

$$r_1 = \frac{15 \sin 120}{\sin 150} = 15\sqrt{3} \text{ gm.wt.}$$

$$r_2 = \frac{15 \sin 90}{\sin 150} = 30 \text{ gm.wt.}$$

(4) $\overline{AB}$ is a uniform rod of length 100 cm and weight 30 kg.wt. is suspended from its two ends A and B by means of two strings, their other ends are fixed at a pin in the ceilling at the point C, if the two strings are perpendicular and AC = 50 cm. Find the tension in each of the two strings.

Solution:



$\because AC = \frac{1}{2} AB$, $m(\angle C) = 90°$

$\therefore m(\angle B) = 30°$

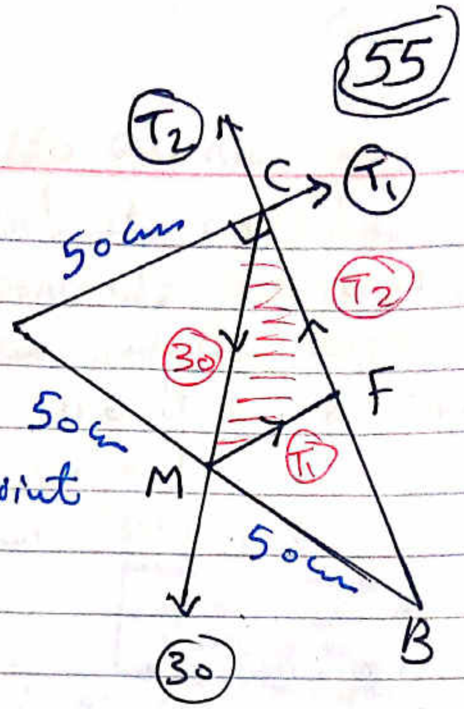
by using Lami's rule:

$$\frac{T_1}{\sin 150} = \frac{T_2}{\sin 120} = \frac{30}{\sin 90}$$

$$T_2 = \frac{30 \sin 120}{\sin 90} = 15\sqrt{3} \text{ kg.wt.}$$

$$T_1 = \frac{30 \sin 150}{\sin 90} = 15 \text{ kg.wt}$$

Another solution



$\because$ M is a midpoint of $\overline{AB}$

, $\overrightarrow{MF} \parallel \overrightarrow{AC}$ $\therefore$ F is a midpoint of $\overline{BC}$

$\therefore MF = \frac{1}{2} AC = 25$ cm

, $FC = \frac{1}{2} BC = 25\sqrt{3}$ cm , $CM = \frac{1}{2} \overline{AB} = 50$ cm

$\therefore \Delta$ CFM is the triangle of forces.

$$\therefore \frac{T_1}{MF} = \frac{T_2}{CF} = \frac{30}{CM}$$

$$\therefore \frac{T_1}{25} = \frac{T_2}{25\sqrt{3}} = \frac{30}{50}$$

$$\Rightarrow T_1 = \frac{30 \times 25}{50} = 15 \text{ kg.wt}$$

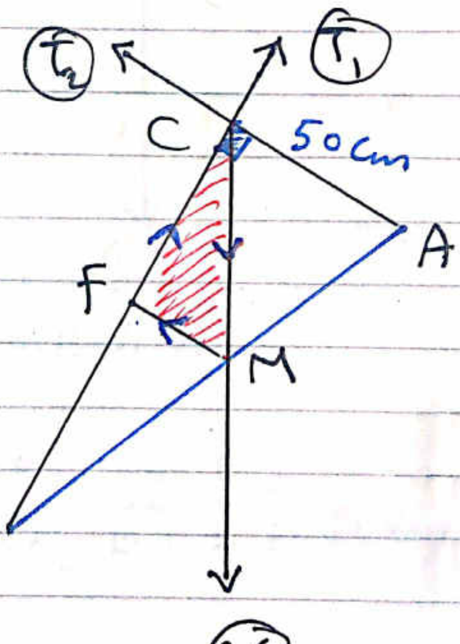
$$T_2 = \frac{25\sqrt{3} \times 30}{50} = 15\sqrt{3} \text{ kg.wt}$$

5 A uniform rod of length 130 cm. and weight 26 newton is suspended at its ends by two strings tied at one point. If the length of one of them is 50 cm. and the length of the other one is 120 cm. what is the position in wich the rod is equilibrium and what is the tension in each of two strings?

Solution:

∵ The lines of action of T_1, T_2 meeting at the point C

∴ the action line of the weight of the rod Should by pass through C also.



The Δ CMF is the triangle of forces

∵ M is midpoint of $\overline{AB}$

, $\overrightarrow{MF} \parallel \overrightarrow{AC}$

∴ F is a midpoint of $\overline{BC}$

∴ $MF = \frac{1}{2} AC = 25$ cm

, $CF = \frac{1}{2} CB = 60$ cm

∵ $(50)^2 + (120)^2 = (130)^2$

∴ $M(\angle ACB) = 90$ ∴ $CM = \frac{1}{2} AB = 65$

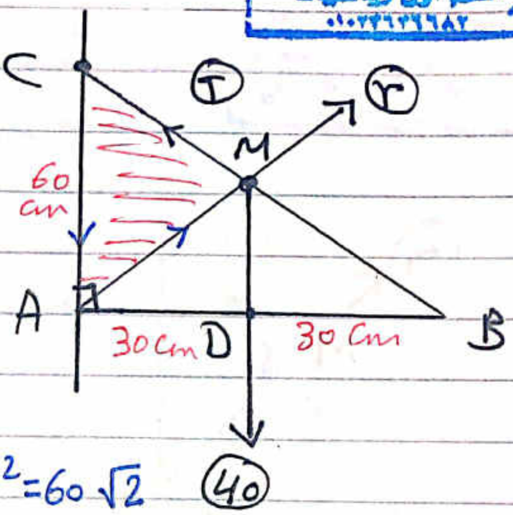
∴ $\frac{T_1}{60} = \frac{T_2}{25} = \frac{26}{65}$ ⇒ $\begin{cases} T_1 = \frac{60\times 25}{65} = 24 \text{ Newton} \\ T_2 = \frac{25\times 26}{65} = 10 \text{ Newton} \end{cases}$

(6) $\overline{AB}$ is a uniform rod with length 60 cm. and weight 40 newton is connected to a hinge on the vertical wall at A. If the rod kept in equilibrium horizontally by a light string connected to the rod B and with the point C on the wall just above A and at a distance 60 cm. from A. Find the tension on the string and the reaction on the hinge at A.

Solution:

$\because$ The two lines of action of W, T meet at M

$\therefore$ the action line of r passes through M



from ΔABC

$$BC = \sqrt{(60)^2 + (60)^2} = 60\sqrt{2}$$

$\because$ D is a mid point of $\overline{AB}$, $\overline{MD} \parallel \overline{AC}$

$\therefore$ M is a mid point of $\overline{BC}$

$\therefore \Delta AMC$ is the triangle of forces

, $MD = \frac{1}{2} AC = 30$, $AM = \frac{1}{2} BC = 30\sqrt{2}$ cm

$$\therefore \frac{T}{30\sqrt{2}} = \frac{r}{30\sqrt{2}} = \frac{40}{60}$$

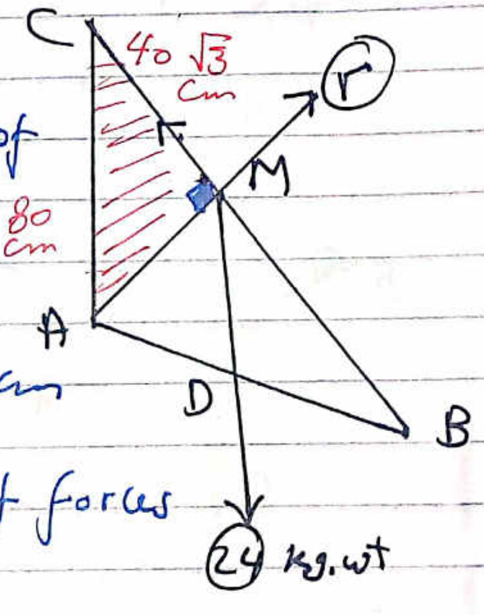
$$\Rightarrow T = r = \frac{40 \times 30\sqrt{2}}{60} = 20\sqrt{2} \text{ newton}$$

7) $\overline{AB}$ is a uniform rod with length 80 cm. and weight 24 kg.wt. acts at its midpoint the end A is attached to a hinge fixed on a vertical wall, and the end B is tied by a light string of lenth $80\sqrt{3}$ cm. fixed at a point C on the wall which lies directly above A and at a distance 80 cm. If the rod is in equilibrium, find the magnitude of the tension and the reaction of the hinge.

Solution

$\because (80)^2 + (80)^2 < (80\sqrt{3})^2$

$\therefore \angle CAB$ is an obtuse angle



$\because$ D is a midpoint of $\overline{AB}$

, $MD \parallel AC$ $\therefore$ M a midpoint of $\overline{BC}$

, $\because AB = AC$ $\therefore \overline{AM} \perp \overline{BC}$

$\therefore AM = \sqrt{(80)^2 - (40\sqrt{3})^2} = 40$ cm

$\therefore \Delta AMC$ is a triangle of forces

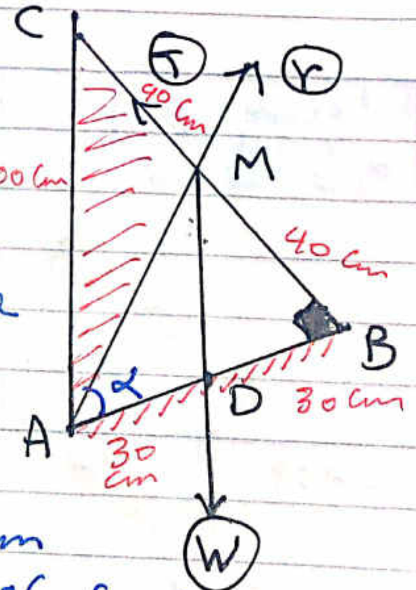
$\therefore \dfrac{T}{40\sqrt{3}} = \dfrac{r}{40} = \dfrac{24}{80}$

$\Rightarrow T = \dfrac{40\sqrt{3} \times 24}{80} = 12\sqrt{3}$ kg.wt.

, $r = \dfrac{40 \times 24}{80} = 12$ kg.wt.

8) $\overline{AB}$ is a uniform rod of length 60 cm. and weight (W) kg.wt. The end A is attached to a hinge fixed on a vertical wall and the end B is tied by a string of length 80 cm., its other end is fixed to a point on the wall vertically above A directly and at a distance 100 cm. of it, then the rod became in equilibrium. Find the tension in the string and the reaction of the hinge, also find the measure of the angle of inclination of the reaction of the hinge to the rod.

Solution:



$\because (80)^2 + (60)^2 = (100)^2$

$\therefore m(\angle ABC) = 90^\circ$

$\therefore \angle BAC$ is acute angle

from $\triangle ABM$

$AM = \sqrt{40^2 + 60^2} = 20\sqrt{13}$ cm

$\triangle AMC$ is the triangle of forces.

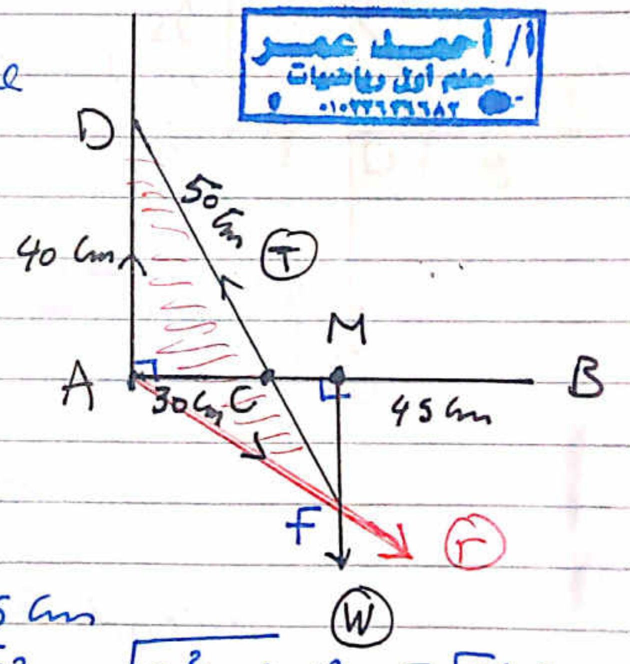
$\therefore \frac{T}{40} = \frac{r}{20\sqrt{3}} = \frac{W}{100} \Rightarrow \begin{cases} T = \frac{2}{5} W \text{ kg.wt.} \\ r = \frac{\sqrt{13}}{5} W \text{ kg.wt.} \end{cases}$

$\tan\alpha = \frac{40}{60} \Rightarrow \alpha = 33^\circ 41' 24''$

9) $\overline{AB}$ is a uniform rod of length 90 cm, and weight (W) kg.wt. Its end A is fixed to a vertical wall by a hinge and the rod is kept in equilibrium horizontally by means of a string of length 50 cm. One of its ends is tied at the point C on the rod at a distance 30 cm from A, the other end of the string is fixed at a point D on the vertical wall above A directly, calculate the tension in the string and the reaction of the hinge on the rod.

Solution:

ΔAFD is the triangle of forces:



$\because \overrightarrow{MF} \parallel \overline{AD}$

$\therefore \Delta CMF \sim \Delta CAD$

$\therefore \frac{CM}{CA} = \frac{MF}{AD} = \frac{FC}{CD}$

$\Rightarrow \frac{15}{30} = \frac{MF}{40} = \frac{FC}{50}$

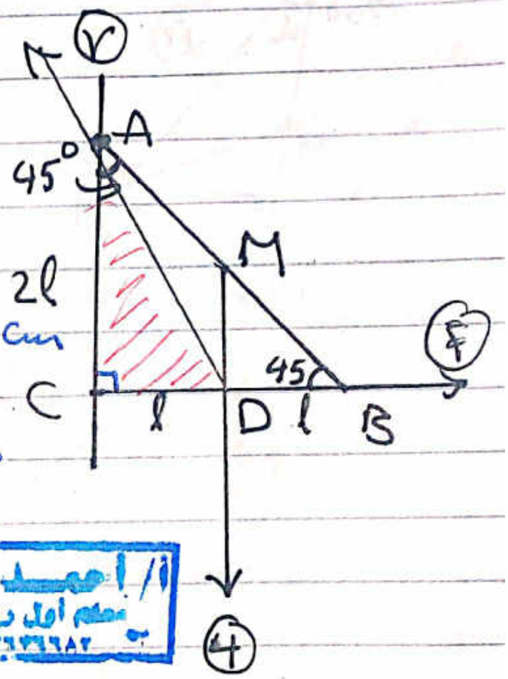
$\Rightarrow MF = 20$ cm, $FC = 25$ cm

$\Rightarrow AF = \sqrt{(MF)^2 + (AM)^2} = \sqrt{20^2 + (45)^2} = 5\sqrt{97}$ cm

$\therefore \frac{T}{50+25} = \frac{r}{5\sqrt{97}} = \frac{W}{40} \Rightarrow \begin{cases} T = \frac{W \times 75}{40} = \frac{15}{8} W \text{ kg.wt.} \\ r = \frac{5\sqrt{97}\, W}{40} = \frac{\sqrt{97}}{8} W \text{ kg.wt} \end{cases}$

10) $\overline{AB}$ is a uniform rod, its end A is attached by a hinge fixed in a vertical wall. A horizontal force acts at the end B to keep the rod in equilibrium while it is inclined to the wall by an angle of measure 45°. If the weight of the rod is 4 kg.wt. acts at its midpoint, then find the magnitude of the force and the reaction of the hinge.

Solution:



Δ ACD is the triangle of forces

Let $AC = 2l \Rightarrow CB = 2l \therefore CD = l$ cm

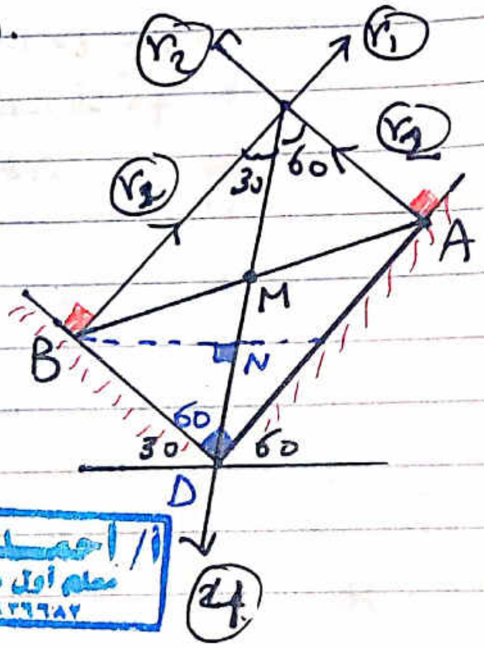
$AD = \sqrt{(2l)^2 + (l)^2} = \sqrt{5}\, l$ cm

$\therefore \frac{F}{l} = \frac{r}{\sqrt{5}\, l} = \frac{4}{2l}$

$\Rightarrow F = \frac{4l}{2l} = 2$ kg.wt.

$r = \frac{4 \times \sqrt{5}\, l}{2l} = 2\sqrt{5}$ kg.wt.

11) A uniform rod of weight 4 newton is placed on two smooth planes inclined at $30°$ and $60°$ to the horizontal. Find the magnitude of the pressure on each plane and the measure of the angle of inclimation, inclinimation of the rod to the horizontal in state of equilibrium.



Solution,

$\because$ the two planes are smooth

$\therefore r_1 \perp \overline{AD}$, $r_2 \perp \overline{BD}$

from lami's rule

$$\frac{r_1}{\sin 120} = \frac{r_2}{\sin 150} = \frac{4}{\sin 90}$$

$$\Rightarrow r_1 = \frac{4 \sin 120}{\sin 90} = 2\sqrt{3} \text{ newton}$$

$$r_2 = \frac{4 \sin 150}{\sin 90} = 2 \text{ newton}$$

$\because MB = MD$ $\therefore \triangle MBD$ is equilateral triangle

$\therefore m(\angle NBD) = 30$

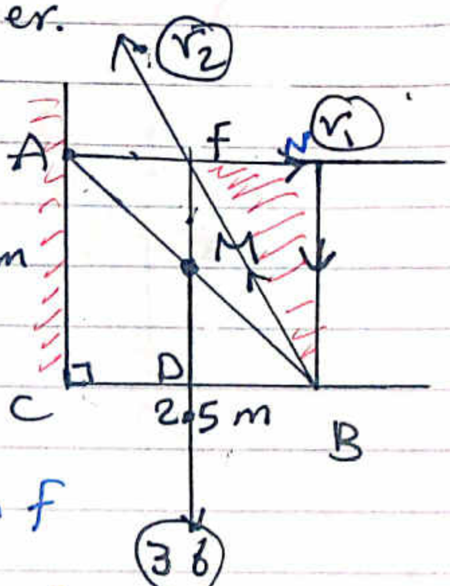
$\therefore$ the measure of the inclimimation of the rod to the horizontal is $30°$

Note that: $P_1 = r_1 = 2\sqrt{3}$ Newton

$P_2 = r_2 = 2$ newton

12 $\overline{AB}$ is a uniform Ladder of weight 36 kg.wt. rest at the end A against a vertical smooth wall, and the other end B on a horizontal rough ground. If the ladder is in equilibrium when its end A is at a distance 3 metres from the ground and the end B is at a distance 2.5 metres from the wall. Find the reaction of each of the ground and the wall on the ladder.

Solution:



$\because$ the wall is smooth

$\therefore \vec{r_1} \perp \overline{AC}$

$\because$ the two lines of action of r_1, 36 meeting at f

$\therefore \vec{r_2}$ should passes through f

$\therefore \Delta$ BFN is the triangle of forces

, $NF = \frac{1}{2} CB = 1.25$ m , $NB = AC = 3$ m

$\therefore FB = \sqrt{(1.25)^2 + (3)^2} = 3.25$ cm

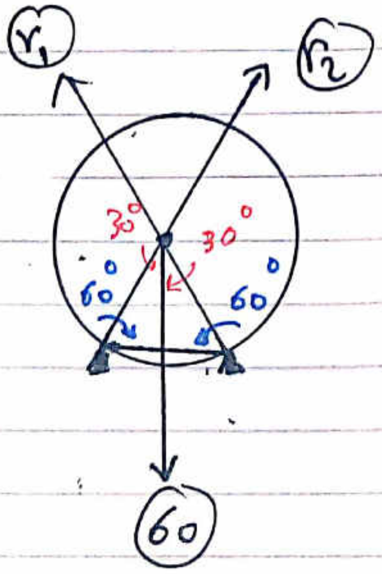
$\therefore \frac{r_1}{1.25} = \frac{r_2}{3.25} = \frac{36}{3}$

$\therefore r_1 = \frac{36 \times 1.25}{3} = 15$ kg.wt. , $r_2 = \frac{36 \times 3.25}{3} = 39$ kg.wt

13 A homogeneous sphere rests on two parallel rods lie on the same horizontal plane The distance between them equals the radius length of the sphere. Find the pressure on each rod if the weight of the sphere is 60 newton.

Solution:

by using Lami's rule



$$\frac{r_1}{\sin 150} = \frac{r_2}{\sin 150} = \frac{60}{\sin 60}$$

$$\therefore r_1 = r_2 = \frac{60 \sin 150}{\sin 60}$$

$$= 20\sqrt{3} \text{ newton}$$

$$\therefore P_1 = r_1 = 20\sqrt{3} \text{ newton}$$

$$, P_2 = r_2 = 20\sqrt{3} \text{ newton}$$

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